PROPAGATION OF ACOUSTIC-GRAVITY WAVES IN INHOMOGENEOUS OCEAN ENVIRONMENT GENERATED BY SEA BOTTOM DEFORMATION

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Abstract. *A coupled-mode system for the propagation of acoustic-gravity waves in inhomogeneous ocean basins is presented. One application of such simulation involves the prediction of acoustic-gravity waves, generated by extended, seismic induced dislocations/landslides, associated with tsunamis. Assuming a certain degree of ocean compressibility, the water wave model equations are enhanced in order to predict the propagation of acoustic-gravity pulses that accompany tsunami generation. These disturbances travel at a much greater speed than the free-surface modes and may be employed as early warning signals. The present work extends closed form solutions, presented for homogeneous, flat bottom environments, subjected to localized seabed dislocation, into a multi-layered ocean configuration. In pursue of more realistic models, the proposed system of equations incorporates variable bathymetry and interfaces, and accounts for diffraction and shoaling effects along with refraction index variability. The derivation of the coupled mode system of equations is based on the construction of local vertical bases along the horizontal domain and the representation of the solution as an enhanced local-mode expansion. The series include additional terms for the consistent treatment of variable interfaces. The extra modes result in convergence acceleration for the modal series, making the method robust and easily extendable to 3-D environments.*

1 INTRODUCTION

The compressibility assumption for the ocean waveguide renders the propagation of acoustic-gravity waves possible for any given frequency ω . In the low frequency spectrum, ocean waves generate acoustic modes as a result of the non-linear interaction of two opposing pairs of gravity waves with equal or similar frequencies. The acoustic –gravity mode excitation occurs due to the non-linearity of the hydrodynamic equations [1-2]. In [3] the non-linear triad interaction between the gravity wave pair and their resulting acoustic gravity wave in presented. Acoustic –gravity waves are also generated by seismic activity in the seabed [4-5], submarine explosions and tsunami wave propagation [4, 6]. As they propagate away from the open ocean towards the shore, acoustic-gravity modes interact with the layers of the inhomogeneous ocean wave guide and the variable seabed, undergoing scattering and shoaling effects [7, 8]. The study of scattering effects in an inhomogeneous wave guide finds numerous applications ranging from underwater acoustic propagation and in shallow bathymetries, seismoacoustics [9, 10] and atmospheric acoustics [11]. As acoustic modes travel considerably faster than the gravity mode, their study finds an immediate application is early tsunami warning systems with profound practical importance [4, 6].

The generally non-separable scattering initial –boundary value problem can be treated by fully numerical schemes, like finite difference or finite elements, semi-analytical methods, like wave number integration and coupled-mode techniques and asymptotic methods like ray theory and parabolic wave models, i.e. [10, 12].

In the present work, a fast-convergent spectral model is presented for treating the incident wave propagation and scattering problems, generated by a vertically oscillating seabed, in stratified, non-uniform waveguides governed by the Helmholtz equation. The proposed method is based on a local mode series expansion, derived by means of local eigenfunction systems defined through the solution of vertical eigenvalue problems, formulated along the waveguide. The local mode series is enhanced by including an additional mode accounting for the inhomogeneous waveguide boundaries and/or interfaces [13-15]. After presenting the initial boundary value problem in Sect. 2 of the present contribution and examining the representation of the oscillating seabed in Sect. 3, the enhanced coupled-mode system is implemented in Sect. 4. A series or numerical results are obtained are presented in Sect. 5 for a given two layered ocean waveguide.

2 PROBLEM DESCRIPTION

We consider acoustic-gravity waves propagating in the multilayered ocean waveguide of Fig.1. For simplicity we restrict ourselves to a 2D problem in Cartesian coordinates, governed by the Helmholtz equation. However, the present method and analysis can be naturally extended to general 3D acoustic-gravity waveguides. The domain $D = D^{(1)} \cup D^{(2)} \cup D^{(3)}$ is decomposed into three parts $D^{(m)}$, $m = 1,2,3$ (see Fig. 1), as follows: $D^{(1)}$ is the subdomain characterized by $x_1 < a$ and $D^{(3)}$ is the subdomain characterized by $x_1 > b$ $(b > a)$, and $D^{(2)}$ is the variable cross section subdomain lying between $D^{(1)}$ and $D^{(3)}$. A similar decomposition is also applied to the (upper and lower) boundaries, as well as to the internal interfaces. The acoustic medium inside the domain is stratified. The physical properties of the layers, vary with respect to the (x,z) coordinates in the middle range-dependent subdomain $D^{(2)}$, and present only vertical variability in the two semi-infinite subdomains $D^{(1)}$ and $D^{(3)}$. Assuming that the whole domain consists of M layers, a total number of $M-1$ interfaces at $z = -h_j(x)$, $j = 1,2,...,M-1$, are considered, where $h_j(x)$ denotes the local depth of each interface (see Fig.1). The waveguide is terminated below by a perfectly rigid (acoustically hard) horizontal boundary, located at $z = -H$. Finally, the waveguide is terminated above by an acoustically soft boundary, located at $z = 0$, corresponding to the free surface.

The density ρ_j, $j = 1,2,..M$, of each layer is assumed to be constant within the layer, presenting possibly sharp discontinuities at the interfaces. Moreover, the sound speed $c_j(x,z)$, $j = 1,2,..M$, presents both vertical and horizontal variability in the middle subdomain $D^{(2)}$, and could also exhibit strong discontinuity at the interfaces. The sound speed becomes function only of z in the two semi-infinite subdomains $D^{(1)}$ and $D^{(3)}$, which are then rendered independent subdomains with respect to both geometry and physical parameters. This fact permits us to obtain complete expansions of the wave field in the semi-infinite regions by means of separation of variables, and consistently formulate the conditions of wave incidence and transmission at *x*=*a* and *x*=*b*, respectively.

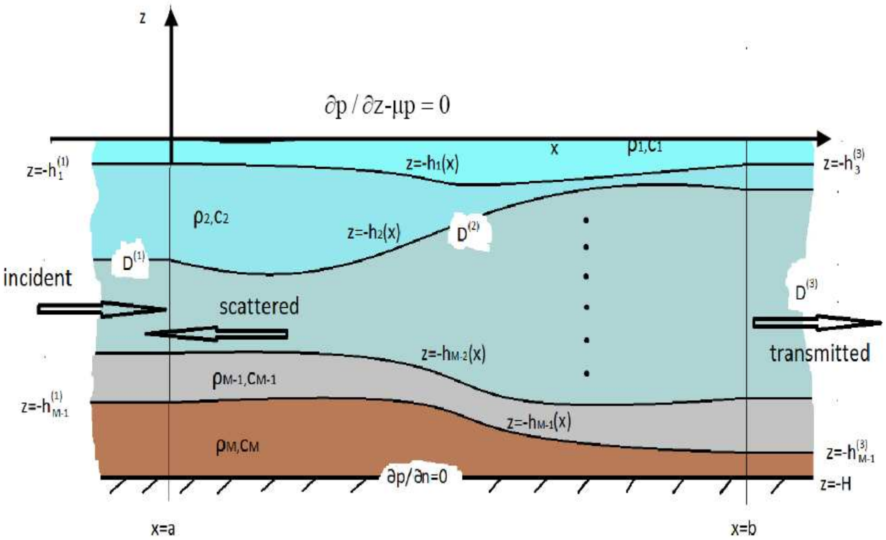


Figure 1. Multilayered two-dimensional acoustic waveguide.

2.1 Governing Equations

Restricting ourselves to monochromatic waves of angular frequency *ω=2π f*, the acoustic-gravity harmonic wave propagation problem inside the present multi-layered waveguide is governed by the Helmholtz equation. The respective boundary value problem takes the form of finding the continuous function p representing the acoustic pressure such that

$$\nabla \cdot \left(\frac{1}{\rho} \nabla p \right) + \frac{k^2}{\rho} p = 0 \ , \qquad \text{in} \quad D \quad , \tag{1}$$

where the wavenumber $k(x,z)=\omega/c(x,z)$ is a piecewise smooth function of the spatial coordinates, possibly presenting sharp discontinuities at the interfaces $z=-h_j(x),\quad j=1,2,...,M-1$. Eqn (1) is supplemented by the following boundary conditions

$$\partial p/\partial z-\mu p=0\,,\quad \text{on the free surface}\quad z=0\,, \tag{2}$$

$$\partial p/\partial n=\partial p/\partial z=0\,,\quad \text{on the perfectly rigid (terminating) boundary at}\quad z=-H\,, \tag{3}$$

where $\mu=\omega^2/g$ is the frequency parameter of gravity waves (g=9.81m/s^2), in conjunction with the interface conditions

$$\frac{1}{\rho_j}\frac{\partial p}{\partial n}=\frac{1}{\rho_{j+1}}\frac{\partial p}{\partial n}\quad \text{on}\quad z=-h_j(x),\quad j=1,2,...,M-1\,. \tag{4}$$

In the previous equations $\partial p/\partial n=\mathbf{n}\nabla p$ denotes the normal derivative, where **n** is the unit normal vector on each boundary and interface. We consider a transmission problem forced by plane waves propagating in the positive x direction. The waves are incident from $D^{(1)}$, and then they are refracted and scattered in the range dependent subdomain $D^{(2)}$, and finally transmitted in $D^{(3)}$. In order to treat the present problem in the infinite domain, complete normal-mode type representations of the wave field in the regions of incidence $D^{(1)}$ and transmission $D^{(3)}$ are derived by separation of variables. In particular, the expansion of the wavefield in $D^{(1)}$ consists of incident and reflected (scattered) waves is as follows,

$$p^{(1)}=p^{(I)}+p^{(R)}=\sum_{n=1}^{\infty}\left(A_n^{(1)}e^{ik_n^{(1)}x}+B_n^{(1)}e^{-ik_n^{(1)}x}\right)Z_n^{(1)}(z)\,, \tag{5}$$

where $p^{(I)}\left(x,z\right)$ stands for the incident and $p^{(R)}\left(x,z\right)$ for the reflected/diffracted component back-propagating in $D^{(1)}$. The functions $Z_n^{(1)}(z)$ and the numbers $k_n^{(1)},\, n=1,2,3...,$ satisfy the following vertical eigenvalue problem in $D^{(1)}$

$$\frac{d^2Z_n^{(1)}}{dz^2}+\left[\left(k^{(1)}\left(z\right)\right)^2-(k_n^{(1)})^2\right]Z_n^{(1)}=0\,, \tag{6}$$

$$\frac{dZ_n^{(1)}(z=0)}{dz}-\mu Z_n^{(1)}(z=0)=0,\qquad \frac{dZ_n^{(1)}(z=-H)}{dz}=0\,, \tag{7a,b}$$

in conjunction with the interface condition

$$Z_n^{(1)}(-h_j^{(1)}+0)=Z_n^{(1)}(-h_j^{(1)}-0),\quad j=1,2,\ M-1\quad , \tag{8}$$

$$\frac{1}{\rho_j}\frac{\partial Z_n^{(1)}(-h_j^{(1)}+0)}{\partial z}=\frac{1}{\rho_{j+1}}\frac{Z_n^{(1)}(-h_j^{(1)}-0)}{\partial z},\quad j=1,2,\ M-1\,, \tag{9}$$

where $k^{(1)}\left(z\right)=\omega/c^{(1)}\left(z\right)$. Similarly, the expansion of the acoustic wavefield in the region of transmission $D^{(3)}$, consists only of outgoing radiated waves, and is given by

$$p^{(3)}=\sum_{n=1}^{\infty}\left(A_n^{(3)}e^{ik_n^{(1)}x}\right)Z_n^{(3)}(z)\,, \tag{10}$$

where the eigenfunctions $Z_n^{(3)}(z)$ and the corresponding eigenvalues $k_n^{(3)},\quad n=1,2,3...,$ are obtained by vertical eigenvalue problems formulated in $D^{(3)}$, as shown above. From the properties of regular Sturm-Liouville problems ([17], [18]) the eigenvalues $\left\{\left(k_n^{(m)}\right)^2, n=1,2..\right\}$, m=1,3, are discrete, infinite, with continuously decreasing moduli, and thus, the corresponding parameters $\left\{k_n^{(m)},\ n=1,2,3...\right\}$, are subdivided into a finite real

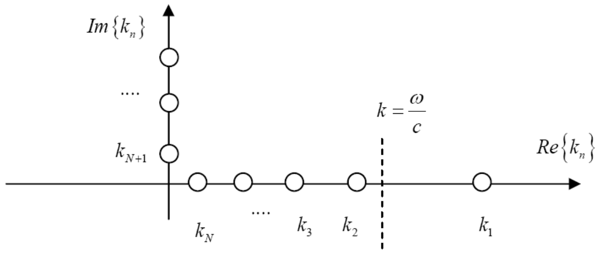


Figure 2. Distribution of eigenvalues on the complex plane.

subset $\left\{k_n^{(m)},\ n=1,2,3...N_p^{(m)}\right\}$ and an infinite imaginary one $\left\{i\left|k_n^{(m)}\right|,\ n=N_p^{(m)}+1,....\right\}$, where $N_p^{(m)}$, denotes the number of propagating modes in $D^{(m)}$, $m=1,3$. The first eigenvalue (n=1) and corresponding eigenvector is essentially associated with the free-surface gravity mode which presents fast decay in depth. Details concerning the exact analytical solution of this problem in the case of 2 layers are given in the APPENDIX, while the distribution of eigenvalues is shown in Fig.2. In order for the wave field to remain bounded at infinity, the coefficients of the expansion $A_n^{(1)}=0,\ n>N_p^{(1)}$. On the other hand the terms $A_n^{(1)}\exp\left(ik_n^{(1)}x\right)Z_n^{(1)}(z)$, $n\le N_p^{(1)}$, constitute the given data associated the incident wave field,

$$p^{(I)}(x,z)=\sum_{n=1}^{N_p^{(1)}}A_n^{(1)}e^{ik_n^{(1)}x}Z_n^{(1)}(z). \tag{11}$$

The above terms could be separately considered as forcing of the present waveguide and the solutions can be obtained by superposition of the responses. In addition, the terms $\exp\left(-ik_n^{(1)}x\right)Z_n^{(1)}(z)$, $n>N_p^{(1)}$, and $\exp\left(ik_n^{(3)}x\right)Z_n^{(3)}(z)$, $n>N_p^{(3)}$, are the evanescent modes in $D^{(m)}$, $m=1,3$, respectively. These modes decay exponentially at large distances from the inhomogeneity in the two semi-infinite strips. By exploiting the representations (5) and (10), the problem can be formulated as a transmission boundary value problem in the bounded subdomain $D^{(2)}$, satisfying eqs (1), (2) (3) and (4) and the following matching conditions:

$$p^{(2)}(x,z)=p^{(1)}(x,z),\qquad \frac{\partial p^{(2)}}{\partial x^{(2)}}=\frac{\partial p^{(1)}}{\partial x^{(1)}},\qquad x=a,\quad -H<z<0, \tag{12a}$$

$$p^{(2)}(x,z)=p^{(3)}(x,z),\qquad \frac{\partial p^{(2)}}{\partial x}=\frac{\partial p^{(3)}}{\partial x},\qquad x_1=b,\quad -H<z<0. \tag{12b}$$

3 GENERATION OF ACOUSTIC-GRAVITY WAVES BY BOTTOM OSCILLATION

The examined problem is supplemented by information concerning the amplitudes $A_n^{(1)}$, $n=1,2,..N_p^{(1)}$, of the incident acoustic-gravity waves entering the ocean waveguide and propagating in the inhomogeneous region. In the case of waves generated by sea-bottom deformation as shown in Fig.3, a simplified model proposed by Yamamoto [5] and used by other authors (see, e.g.., Stiassnie [4]), is based on the oscillatory motion of a block of width 2c of the ocean floor with vertical (piston motion) amplitude a_0, and frequency ω. In this case the excitation of the acoustic gravity wave field is shown to be represented by the induced field generated by two symmetric line sources located at the depth of the interface modelling the rigid sea floor, indicated by open circles in Fig.3. In this case the excitation is given by (Boyles [9], Jensen et al [10])

$$p^{(I)}(x,z)=2\rho\omega^2\alpha_0\sum_{n=1}^{\infty}\frac{Z_n^{(1)}(z)Z_n^{(1)}(z_0)}{k_n^{(1)}\left\|Z_n^{(1)}\right\|^2}\left(\exp\left(ik_n^{(1)}\left|x+c\right|\right)+\exp\left(ik_n^{(1)}\left|x-c\right|\right)\right), \tag{13}$$

where z_0 denotes the depth of the line source. The latter parameter can be further tuned for better fitting the data concerning the free-surface elevation generated by large extent seismic excitation of the seabed at large depths.

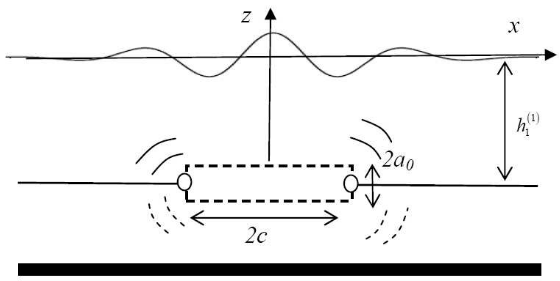


Figure 3. Wave generation model by oscillatory bottom dislocation

The above expression simplifies at some distance from the origin by omitting the evanescent modes $n > N_p^{(1)}$, and thus at the entrance of the inhomogeneous acoustic-gravity waveguide $x = a$ it holds

$$A_n^{(1)}(\omega) = 2\rho\omega^2\alpha_0 \frac{Z_n^{(1)}(z_0)}{k_n^{(1)}\left\|Z_n^{(1)}\right\|^2}\left(\exp\left(ik_n^{(1)}(x+c)\right)+\exp\left(ik_n^{(1)}(x-c)\right)\right) =$$
$$= 2\rho\omega^2\alpha_0 \frac{Z_n^{(1)}(z_0)}{k_n^{(1)}\left\|Z_n^{(1)}\right\|^2}\cos\left(k_n^{(1)}c\right)\exp\left(ik_n^{(1)}a\right), \tag{14}$$

It is worth mentioning here that although the above model corresponds to simple harmonic excitation and it could be further extended to treat multichromatic excitation, as well as to treat the linearized time-domain problem where the motion starts from rest (see, also Stiassnie [4]).

4 PROPAGATION OF ACOUSTIC-GRAVITY WAVES BY COUPLED MODE SYSTEM

4.1 Enhanced local-mode representation

Inside the bounded domain $D^{(2)}$, the solution $p(x,z)$ may be set into a standard spectral-type representation, based on local-mode series, as follows

$$p\ (x,z) = \sum_{n=1}^{\infty} U_n(x) Z_n(z;x)\ . \tag{15}$$

The family of local vertical basis functions $Z_n(z;x)$ appearing in the above expansion, which are parametrically dependent on x, is obtained by formulating and solving local, vertical Sturm-Liouville problems in the z-intervals $[-H,\, 0]$, for each horizontal position $a<x<b$. However, any finite truncation of the series (15) is incompatible with any of the sloping interface conditions, whenever $dh_j(x)/dx \neq 0$, $j = 1,2,...M-1$, rendering the above series to converge only in an L^2-sense, and the coefficients U_n to decay like $O\left(n^{-2}\right)$; see Belibassakis et al [13]. To remedy this inconsistency, an additional mode associated with each interface is introduced, denoted by $U_j(x)Z_j(z;x)$, $j = -M+2,..,-1,0$. These modes are called sloping-interface modes. Thus, we obtain the following enhanced local-mode series

$$p(x,z) = \sum_{n=-M+2}^{0} U_n(x) Z_n(z;x) + \sum_{n=1}^{\infty} U_n(x) Z_n(z;x)\,. \tag{16}$$

The vertical structure of the sloping-interface modes, for every horizontal position $a < x < b$, is any globally continuous and piecewise smooth function defined with support in the local vertical intervals $[-h_{M-1}(x), -h_{M-2}(x)], \ldots, [-h_1(x), 0]$, satisfying the following condition(s)

$$\left.\frac{1}{\rho_j}\frac{\partial Z_n}{\partial z}\right|_{z=-h_j} - \left.\frac{1}{\rho_{j+1}}\frac{\partial Z_n}{\partial z}\right|_{z=-h_j} = 1, \quad j = 1, 2, \ldots, M-1. \tag{17a}$$

Moreover, the function $Z_0(z;x)$ should satisfy the homogeneous Dirichlet condition at $z = \eta(x)$. Consequently, the *M*-1 terms $U_j(x) Z_j(z;x)$, $j = -M+2, .., -1, 0$, are additional degrees of freedom in the bounded subdomain $D^{(2)}$, permitting the consistent satisfaction of all interface conditions. The amplitude of the additional modes is given by

$$U_{j-1}(x) = \left.\frac{1}{\rho_j}\frac{\partial p}{\partial z}\right|_{z=-h_j} - \left.\frac{1}{\rho_{j+1}}\frac{\partial p}{\partial z}\right|_{z=-h_j}, \quad j = 1, 2, ..M-1. \tag{17b}$$

From this last relation, it is evident that no extra mode needs to be introduced in the last layer terminated in the lower flat boundary, where a homogeneous Neumann condition is satisfied.

The important effect of the additional modes is to significantly increase the rate of decay of Z_n – Fourier coefficients of the acoustic wave potential (modal amplitudes). In this case, the modes associated with the enhanced series exhibit a rapid decay rate: $|U_n(x)| \le C(x)\, n^{-4}$, $n \to \infty$, $\forall x \in [a, b]$. The bound $C(x)$ is a continuous function on $[a, b]$ and, thus, the previous estimate is global ; see also [14, 15]. If the additional modes are not included, then the rate of decay of the modes in the standard series (15) is only $|U_n| = O(n^{-2})$. The above result is obtained by means of repetitive use of integration by parts, in conjunction with the properties of the local Sturm-Liouville system, and will be illustrated through appropriate numerical examples in a subsequent section.

4.2 The Coupled-Mode System (CMS)

Having obtained the eigenfunctions associated with the local vertical problem, $\forall x \in [a, b]$, we proceed to the calculation of the mode amplitudes $\{U_j(x),\ j = -M+2, .., -1, 0, 1, 2, ...\}$. To this respect we substitute the enhanced local mode representation (16) in the variational principle of the transmission problem (see Athanassoulis & Belibassakis [14], and express the variation of the unknown field $p(x,z) \in D^{(2)}$, through the variations of the modal amplitudes

$$\delta p(x,z) = \sum_{n=-M+1}^{\infty} Z_n(z;x)\, \delta U_n(x) \quad . \tag{18}$$

Next, by considering only the variations $\delta U_n(x)$, $n = -M+2, .., 0, 1, \ldots$, in $a < x < b$, and using in the variational principle of the problem (Belibassakis et al [19]) we obtain the following coupled-mode system (CMS) of second-order ordinary differential equations, with respect to the mode amplitudes, $U_n(x), n = -M+2, .., 0, 1, 2, ..$,

$$\sum_{n=-M+2}^{\infty} a_{mn}(x)\frac{d^2 U_n(x)}{dx^2} + b_{mn}(x)\frac{dU_n(x)}{dx} + c_{mn}(x) U_n(x) = 0, \tag{19}$$

where $m = -M+2,..,0,1,2,...$ The x-dependent coefficients a_{mn}, b_{mn} and c_{mn} of the present CMS (19) are defined in terms of $Z_n(z;x)$ in $a < x < b$ and are given by

$$a_{mn} = \langle Z_n, Z_m \rangle , \tag{20}$$

$$b_{mn} = 2\left\langle \frac{\partial Z_n}{\partial x}, Z_m \right\rangle + \sum_{j=1}^{M-1} \left(\frac{1}{\rho_j} - \frac{1}{\rho_{j+1}} \right) \frac{dh_j}{dx} Z_n(-h_j) Z_m(-h_j) , \tag{21}$$

$$c_{mn} = \left\langle \frac{\partial^2 Z_n}{\partial x^2} + \frac{\partial^2 Z_n}{\partial z^2} + k^2 Z_n, Z_m \right\rangle + \sum_{j=1}^{M-1} \left([\![Z_{n,z}]\!] + \frac{dh_j}{dx} [\![Z_{n,x}]\!] \right) Z_m(-h_j) . \tag{22}$$

In the above relations, $\langle f, g \rangle := \int_{-H}^{\eta} \rho^{-1} f(z) g(z) dz$ is the weighted inner product of $L^2(-H,0)$ function spaces, for all x in $a < x < b$. Further, the quantities $[\![Z_n]\!]_x$ are defined by

$$[\![Z_{n,w}]\!] = \left[\frac{1}{\rho_j} \frac{\partial Z_n}{\partial w}\bigg|_{z=-h_j^+} - \frac{1}{\rho_{j+1}} \frac{\partial Z_n}{\partial w}\bigg|_{z=-h_j^-} \right], \quad j = 1,2,...,M-1 . \tag{23}$$

From the variational equation (see Belibassakis et al [14]), defined on the vertical interfaces at $x = a$ and $x = b$, respectively, we obtain the following end-conditions for the mode amplitudes $U_n(x)$,

$$C_n^{(m)} dU_n / dx_1 + D_n^{(m)} U_n = F_n^{(m)}, \quad n = 0,1,2,... , \quad m = 1,3 , \tag{24}$$

where the coefficients $C_n^{(m)}, D_n^{(m)}, m = 1,3$, are defined in terms of the physical parameters at the end points $x = a$ and $x = b$, and

$$F_n^{(1)} = 2iA_n^{(1)}, \; n = 1,..,N_p^{(1)}, \; F_n^{(1)} = 0, \; n > N_p^{(1)}, \quad F_n^{(3)} = 0, \; n = 1,2,3...... , \tag{25}$$

are associated with the forcing of the system, as defined by eqn (14) for various frequencies.

5 NUMERICAL RESULTS AND DISCUSSION

In the present work, the numerical solution of the above coupled-mode system is obtained by truncating the series (16) and using a finite difference scheme based on a uniform grid and second-order central differences to approximate derivatives. The calculation of the spatially varying coefficients is obtained by an hp-FEM for the solution of the Vertical Eigenvalue Problem locally at each horizontal position (Belibassakis et al [19]). In order to further enhance the efficiency of the present model, future work is focused on the application of *p*-Finite Element Methods, in conjunction with grid adaptation techniques based on the spatial variability of the system coefficients a_{mn}, b_{mn} and c_{mn}.

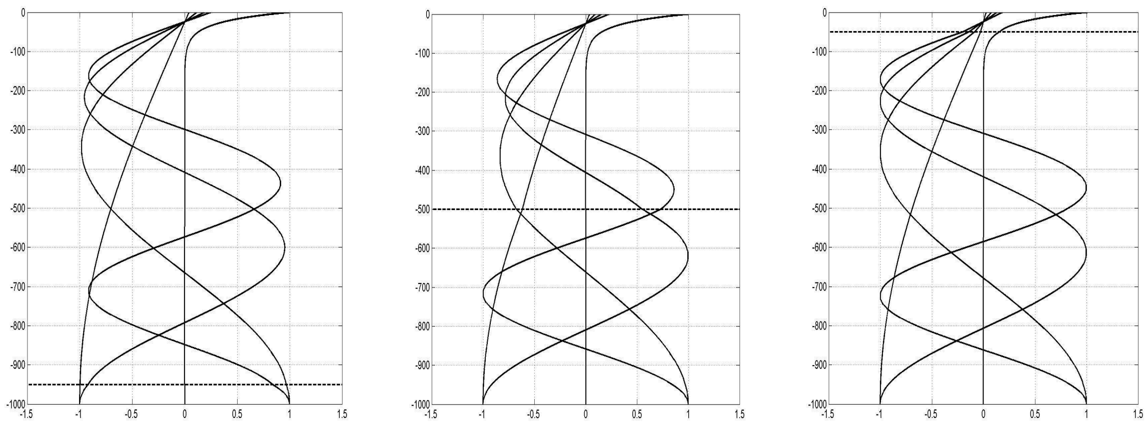
As an example, we consider underwater acoustic propagation in a shoaling environment, characterized by variable seabed boundary. As before, the upper layer (layer 1) is sea water of density and speed of sound $\rho_1 = 1g/cm^3$, $c_1 = 1500\,m/s$. The lower layer (layer 2) corresponds to sand-silt-clay sediment with properties $\rho_2 = 1.5g/cm^3$, $c_2 = 1700\,m/s$, terminated at the impermeable (rigid) bottom which is located at a depth $z = -100m$. The geometry of the internal interface is defined as

$$h_1(x) = 500 - 450\tanh\left[2\pi \left(\left(\frac{x-3000}{4000} \right) - 0.5 \right) \right], \quad a \le x \le b \quad , \tag{26}$$

where *a*=2800m and *b*=7200m. The first 5 eigenfunctions for a relatively low frequency *f*=0.2Hz, at the entrance (*x*=*a*), middle (*x*=5000m) and the exit (*x*=*b*) of the ocean waveguide are presented in Fig.4. The first 5 modes *n*=1,2,3,4,5 are plotted, and the interface separating the upper from the lower medium is shown by using dashed lines. The first eigenvalue (*n*=0) decaying in depth is scaled with respect to its maximum value at the free surface. The rest of eigenvalues (*n*=2,3..) associated with the acoustic modes are scaled with respect to their absolute value at the hard bottom. Numerical results concerning the real part of the calculated wave field, forced by the amplitudes given by eqn (14), when the waveguide is excited at frequency $f = 0.08Hz$ and f=1.5Hz are shown in Fig. 5, as obtained by present method, using 5 totally modes and 501 horizontal equidistant points for the discretization of the CMS. It can be seen that in the low frequency case $f = 0.08Hz$ the gravity mode is excited interacting weakly with the rest of the modes. Near the shallow end of the domain, a small part of the energy is transmitted from the upper medium (water) to the lower medium (sediment). Also, in the case of higher frequency $f = 1.5Hz$ the interaction of the first acoustic mode (n=2) with the gravity mode (n=1) is also very small and the generated free-surface elevation is negligible. Thus, the acoustic pressure is difficult to be observed at the free surface, however this could be possible at lower depths below the free surface. In order to illustrate further this fact we examine the case of a prescribed free surface elevation of the form

$$\eta\left(x,t\right) = \frac{a_0}{\tau} H\left(c^2 - x^2\right) H\left(t\left(\tau - t\right)\right), \tag{27}$$

(see also Stiassnie [4]) where H denotes the Heaviside function. The low frequency-part of the spectrum associated with the above elevation is shown in Fig.6 for τ=1sec. In order to approximately illustrate the effects of this activity concerning wave propagation in the ocean acoustic-gravity waveguide considered, we examine the combined forcing at frequencies f=0.08Hz and f=1.5Hz with amplitudes proportional to the area under the spectrum in the bands [0,1Hz] and [1Hz, 2Hz], respectively, which corresponds to the modelling of the continuous spectrum by summation of delta functions indicated by using arrows in Fig.6



(a) **(b)** **(c)**

Figure 4: First 5 eigenfunctions (n=1,2,3,4,5) in two layer ocean acoustic waveguide with properties $\rho_1 = 1g/cm^3$, $c_1 = 1500\,m/s$ and $\rho_2 = 1.5g/cm^3$, $c_2 = 1700\,m/s$, for frequency f=0.2Hz, and position of the interface at (a) 950m, (b)500m (c) 50m, which is indicated by thick dashed lines.

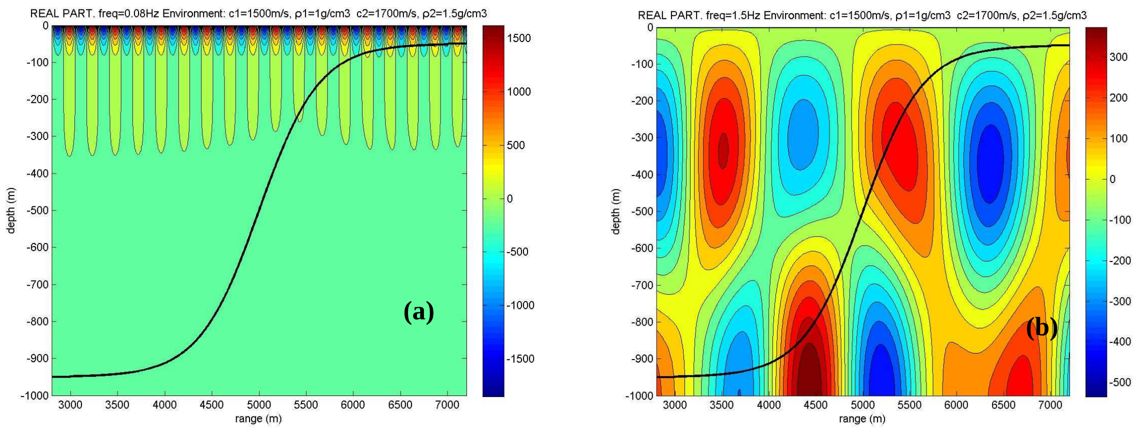


Figure 5. Acoustic pressure (real part) excited by bottom oscillating block ($a_0 = 1m$, $c = 1000m$) at frequency (a) 0.08 Hz and (b) 1.5Hz. The parameters of the ocean waveguide are the same as in Fig.4.

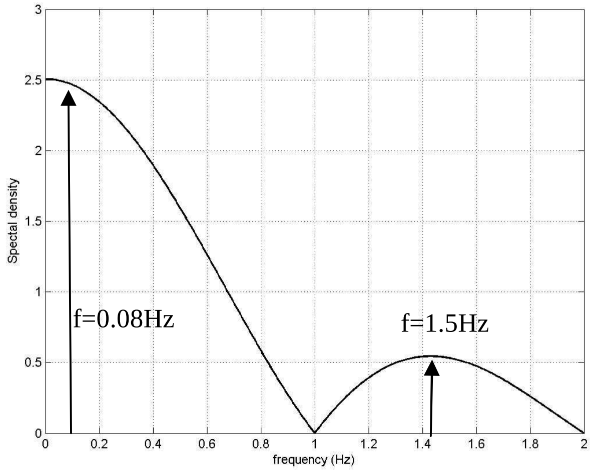


Figure 6. Low frequency part of the spectrum corresponding to free surface elevation eqn (27), for a_0 =1m and duration τ=1sec.

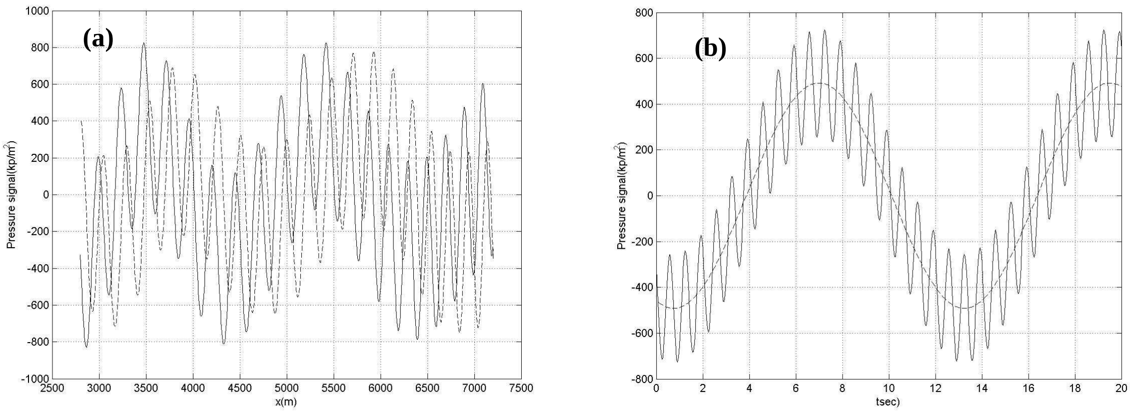


Figure 7. (a) Pressure distribution at depth 50m as approximated from contributions at frequencies f=0.08Hz and f=1.5Hz (indicated by arrows in Fig.5) and (b) Pressure signal at depth 50m at the shallow end of the domain *b*=7200m.

The result is presented in Fig.7 concerning the pressure distribution at depth 50m (location of silty clay seabed in the shallow end of the domain *b*=7200m) as calculated by the present method. For this example the pressure signal at depth 50m at the shallow end of the domain is plotted in the right subplot of Fig.7, where the acoustic-gravity waves associated with the mode *n*=2 are clearly observable as high frequency ripples. The latter, travel at significant higher speed that the free surface waves associated with the gravity leaving an option for an advance warning sufficient for evacuation and protection.

6 CONCLUSIONS

In this work an improved coupled-mode method is presented for the efficient solution of the problem of time-harmonic propagation and scattering of acoustic-gravity waves in a non-uniform stratified waveguide. The problem is governed by the Helmholtz equation, with variable coefficients, in conjunction with the linearized free-surface boundary condition associated with gravity waves. Our method is based on an enhanced local-mode series for the representation of the wave field, including additional modes, accounting for the effects of the inhomogeneous interfaces. In the case of multilayered waveguides, the local vertical eigenvalue problems are treated by *h*- and *p*-FEM, exhibiting robustness and good rates of convergence. In order to further enhance the efficiency of the present model, current work is focused on the application of h*p*-FEM for the solution of the coupled system on the horizontal plane, in conjunction with grid adaptation techniques based on the spatial variability of the system coefficients. Among several other advantages, the present method can be naturally extended to treat wave propagation and scattering problems in 3D multi-layered waveguides.

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APPENDIX

In the case of two layers $1,2$, with constant physical properties $\rho_1, c_1, k_1 = \omega/c_1$ and $\rho_2, c_2, k_2 = \omega/c_2$ respectively, the exact analytical solution of the vertical eigenvalue problem is given by

$$Z_{(1)}(z) = B_n\left[b_n \cos(\lambda_{n1} z) + \sin(\lambda_{n1} z)\right] \quad , \qquad Z_{(2)}(z) = B_n K_n \cos\left[\lambda_{n2}(z+H)\right] \quad , \tag{A1}$$

where,

$$\lambda_{n1} = \sqrt{k_1^{\ 2} - k_n^{\ 2}} \quad , \qquad \lambda_{n2} = \sqrt{k_2^{\ 2} - k_n^{\ 2}} \quad , \tag{A2}$$

and

$$b_n = \frac{\lambda_{n1}}{\mu} \, , \qquad K_n = \frac{b_n \cos(-\lambda_{n1} h_1) + \sin(-\lambda_{n1} h_1)}{\cos\left(\lambda_{n2} h_2\right)} \tag{A3}$$

In this case, the eigenvalues k_n are found as the roots of the equation

$$\frac{\rho_2 \lambda_{n1}}{\rho_1 \lambda_{n2}} \left[b_n \sin(\lambda_{n1} h_1) + \cos(-\lambda_{n1} h_1) \right] = \tan(\lambda_{n2} h_2) \left[\sin(\lambda_{n1} h_1) - b_n \cos(-\lambda_{n1} h_1) \right], \tag{A4}$$

which expresses the continuity of $\rho^{-1} \partial Z / \partial z$ across the interface at $z = -h_1$. The remaining constants B_n, $n = 1, 2, ...$, of the above solution can be fixed by appropriate normalization.