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STATIC ANALYSIS OF THICK LAYERED ANISOTROPIC PLATES WITH BEM

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Abstract. *In this paper the boundary element method (BEM) is developed for the analysis of thick anisotropic plates modeled by Mindlins' theory. Such plates describe the response of laminated plates consisting of layers of anisotropic materials.. The governing equations are three coupled linear partial differential equations of 2nd order subjected to three boundary conditions along the boundary of the plate. The boundary value problem is solved using the analog equation method (AEM). Thus, following the principle of the analog equation the original equations are substituted by three uncoupled Poisson's equations under fictitious loads, which are subsequently solved using the conventional boundary element method for Poisson's equation. On the base of the developed solution procedure, a computer program is written and illustrative plate problems are solved by the proposed method. The obtained numerical results demonstrate the efficiency of the solution procedure, validate its accuracy, and give a revealing insight into the response of the thick laminated plates.*

1 INTRODUCTION

Thick laminated plates consisting of various layers of anisotropic material exhibit many advantages over single layered homogeneous plates and are extensively used in modern structures. Using various laminates of different mechanical properties give the designers the opportunity to increase the strength, minimize the weight and thus optimize the design according to the structural requirements.

As the plate thickness increases the assumptions of the Kirchhoff plate theory are violated and higher order theories, which reduce the 3-dimensional elasticity problem to a 2-dimensinal one, are needed. Mindlin [1] and Reissner [2] were among the first who presented plate theories which take into account the shear deformation of the plate. The Mindlin plate theory combines accuracy and simplicity and can be used for the analysis of moderate thick plates. It assumes linear variation of the displacements across the thickness which implies that a plane cross section remains plane after the deformation. This assumption is in contradiction to the shear effect and a correction coefficient is introduced to overcome the error. The Reissner theory is a higher order theory [3] for shear deformable plates, where the displacement variation is not necessarily linear along the thickness. Apart from the famous Mindlin and Reissner plate theories, there have been presented various higher order theories which can predict accurately the response of thick plates [4]. A systematic procedure in order to obtain various orders of plate theories is presented in [5].

Laminated thick plates are usually studied by Mindlin or higher order plate theories where the displacements are expressed as power series of the transverse coordinate. However, even these theories predict accurately the global response of the laminated plate, they may not ensure the stress continuity between laminates and are inadequate to predict delaminating between two adjacent layers. Various theories which satisfy the interlaminar stress continuity have been presented [6].

Analytic solutions of thick laminated plates have been presented only for simply supported rectangular plates [7-9]. However, solution for realistic plate problems are obtained by numerical methods such as FEM [10-12], differential quadrature method [13] and mesh free methods [14]. The BEM has been presented for the solution of thick isotropic plates using the Mindlin [15] and Reissner plate theory [16,17]. The BEM for orthotropic thick plates has been studied in [18] using the Reissner plate theory.

In this paper we study the bending problem of thick anisotropic plates described by Mindlin's theory. The plate may have arbitrary shape and subjected to domain and boundary loads under any type of admissible boundary conditions. It may consist of several laminates made of anisotropic material. The governing equations are written in terms of the transverse deflection of the middle surface and its rotations about the x and y axis. It is a system of coupled partial differential equations (PDE) of second order, which is solved using the analog equation method [19]. Following the principle of the analog equation the original equations are substituted by three uncoupled Poisson's equation under fictitious loads. The Poisson's equations are solved using the BEM with constant boundary elements and linear triangular elements for domain discretization. Various illustrative plate problems are solved by the proposed method which demonstrate its efficiency, validate its accuracy and give a revealing insight into the response of the thick laminated plates.

2 GOVERNING EQUATIONS

We consider a thick elastic plate made of L layers of uniform thickness occupying the two-dimensional multiply connected domain Ω in the xy-plane with the boundary $\Gamma=\bigcup_{i=0}^{i=K}\Gamma_i$ (Fig. 1). The curves Γ_i $(i=1,2,...,K)$ may be piecewise smooth. The principal axes (x_k,y_k) of each laminate may be inclined with respect to global coordinate system (Fig. 1). The plate may be simply supported, clamped or free along the boundary. The plate is subjected to a transverse surface load $f(x,y)$ in the interior of the domain as well as to transverse loads $Q_n(s)$, bending $M_n(s)$ and twisting moments $M_{nt}(s)$ along the boundary. The Mindlin plate theory is adopted, according to which the displacement field is written as

$$u(x,y,z)=z\phi_x(x,y)\,,\;\; v(x,y,z)=z\phi_y(x,y)\,,\;\; w(x,y,z)=w(x,y) \tag{1a,b,c}$$

where w is the transverse deflection of the middle surface of the plate and ϕ_x,ϕ_y its rotations about y and x axis, respectively. The constitutive equations of the $k=1,2,...,L$ layer, which is made of a general anisotropic material, are written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}^k=\begin{bmatrix}C_{11} & C_{12} & C_{16}\\ C_{12} & C_{22} & C_{26}\\ C_{16} & C_{26} & C_{66}\end{bmatrix}^k\begin{Bmatrix}z\phi_{x,x}\\ z\phi_{y,y}\\ z(\phi_{x,y}+\phi_{y,x})\end{Bmatrix},\;\begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}^k=\begin{bmatrix}C_{55} & C_{45}\\ C_{45} & C_{44}\end{bmatrix}^k\begin{Bmatrix}\phi_x+w_{,x}\\ \phi_y+w_{,y}\end{Bmatrix} \tag{2a,b}$$

where C_{ij} are the elastic constants of the anisotropic material as transformed from the material axes (x_k,y_k) to the global axes (x,y) [20]. Thus, the stress resultants are

$$\begin{Bmatrix}M_x\\ M_y\\ M_{yx}\end{Bmatrix}=\sum_{k=1}^{L}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}^k zdz=\begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}\begin{Bmatrix}\phi_{x,x}\\ \phi_{y,y}\\ \phi_{x,y}+\phi_{y,x}\end{Bmatrix} \tag{3}$$

$$\begin{Bmatrix}Q_x\\ Q_y\end{Bmatrix}=K_s\sum_{k=1}^{L}\int_{z_k}^{z_{k+1}}\begin{Bmatrix}\tau_{xz}\\ \tau_{yz}\end{Bmatrix}^k dz=\begin{bmatrix}A_{55} & A_{45}\\ A_{45} & A_{44}\end{bmatrix}\begin{Bmatrix}\phi_x+w_{,x}\\ \phi_y+w_{,y}\end{Bmatrix} \tag{4}$$

where K_s is the shear correction factor.

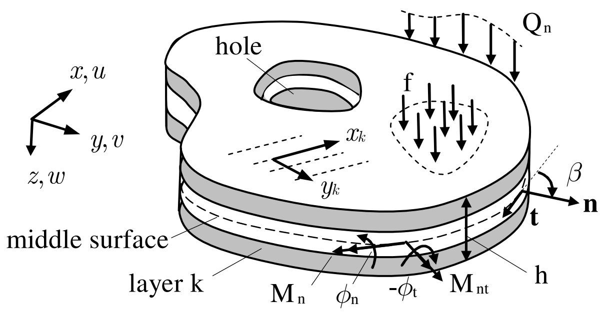


Figure 1. Laminated thick plate

By taking the first variation of the total potential [19] we obtain the equilibrium equations of the problem in terms of the stress resultants

$$\frac{\partial M_x}{\partial x}-\frac{\partial M_{xy}}{\partial y}-Q_x=0 \tag{5a}$$

$$\frac{\partial M_{yx}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y=0 \tag{5b}$$

$$\frac{\partial Q_x}{\partial x}-\frac{\partial Q_y}{\partial y}+f=0 \tag{5c}$$

with the associated boundary conditions

$$a_1 w + a_2 Q_n = a_3 \tag{6a}$$

$$\beta_1\phi_n + \beta_2 M_n = \beta_3 \tag{6b}$$

$$\gamma_1\phi_t + \gamma_2 M_{nt} = \gamma_3 \tag{6c}$$

where

$$Q_n = Q_x n_x + Q_y n_y\,,\; M_n = M_x n_x^2 + M_y n_y^2 + 2n_x n_y M_{yx} \tag{7a,b}$$

$$M_{nt} = M_{xy}(n_y^2 - n_y^2) + n_x n_y (M_x - M_y) \tag{7c}$$

$$\phi_n = n_x\phi_x + n_y\phi_y\,,\; \phi_t = -n_y\phi_x + n_x\phi_y \tag{7d,e}$$

with $n_x = \cos\beta$ and $n_y = \sin\beta$ (Fig. 1). All types of boundary conditions can be derived from Eqs. (6) by appropriate selection of the parameters a_i, β_i, γ_i ($i = 1,2,3$) such as :

(i) clamped for $a_1 = \beta_1 = \gamma_1 = 1$ and $\alpha_2 = \beta_2 = \gamma_2 = \alpha_3 = \beta_3 = \gamma_3 = 0$,

(ii) simply supported of type I (hard) for $a_1 = \beta_2 = \gamma_1 = 1$ and
$\alpha_2 = \beta_1 = \gamma_2 = \alpha_3 = \beta_3 = \gamma_3 = 0$,

(iii) simply supported of type II (soft) for $a_1 = \beta_2 = \gamma_2 = 1$ and
$\alpha_2 = \beta_1 = \gamma_1 = \alpha_3 = \beta_3 = \gamma_3 = 0$ and

(iv) free for $a_2 = \beta_2 = \gamma_2 = 1$ and $\alpha_1 = \beta_1 = \gamma_1 = \alpha_3 = \beta_3 = \gamma_3 = 0$.

Using Eqs. (3)-(4) we obtain the governing equations in terms of the displacements

$$\begin{aligned} D_{11}\phi_{x,xx} + 2D_{16}\phi_{x,xy} + D_{66}\phi_{x,yy} + D_{16}\phi_{y,xx} + (D_{12}+D_{66})\phi_{y,xy} + D_{26}\phi_{y,yy} \\ - A_{55}(\phi_x + w_{,x}) - A_{45}(\phi_y + w_{,y}) = 0 \end{aligned} \tag{8a}$$

$$\begin{aligned} D_{66}\phi_{y,xx} + 2D_{26}\phi_{y,xy} + D_{22}\phi_{y,yy} + D_{16}\phi_{x,xx} + (D_{12}+D_{66})\phi_{x,xy} + D_{26}\phi_{x,yy} \\ - A_{45}(\phi_x + w_{,x}) - A_{44}(\phi_y + w_{,y}) = 0 \end{aligned} \tag{8b}$$

$$A_{55}(\phi_{x,x} + w_{xx}) + A_{45}(\phi_{x,y} + \phi_{y,x} + 2w_{xy}) - A_{44}(\phi_{y,y} + w_{,yy}) = -f \tag{8c}$$

3 AEM SOLUTION

The boundary value problem Eq. (6) and (8) is solved using the AEM [19]. Since the governing Eqs (8) is a system of three coupled second order PDEs, the analog equations are :

$$\nabla^2 w = b_1(x,y)\,,\; \nabla^2\phi_x = b_2(x,y)\,,\;\; \nabla^2\phi_y = b_3(x,y) \tag{9a,b,c}$$

where b_1, b_2 and b_3 are three fictitious sources unknown in the first instance. The solution of Eq. (8a-c) is given in integral form [21]

$$\varepsilon w(\mathbf{x}) = \int_\Omega u^* b_1 d\Omega - \int_\Gamma (u^* w_{,n} - u^*_{,n} w) ds \qquad \mathbf{x} = (x,y) \in \Omega \cup \Gamma \tag{10a}$$

$$\varepsilon \phi_x(\mathbf{x}) = \int_\Omega u^* b_2 d\Omega - \int_\Gamma (u^* \phi_{x,n} - u^*_{,n} \phi_x) ds \qquad \mathbf{x} = (x,y) \in \Omega \cup \Gamma \tag{10b}$$

$$\varepsilon \phi_y(\mathbf{x}) = \int_\Omega u^* b_3 d\Omega - \int_\Gamma (u^* \phi_{y,n} - u^*_{,n} \phi_y) ds \qquad \mathbf{x} = (x,y) \in \Omega \cup \Gamma \tag{10c}$$

in which $u^* = \ell nr / 2\pi$ is the fundamental solution of Eq. (10a); $r = \|\boldsymbol{\xi} - \mathbf{x}\|$, $\mathbf{x} \in \Omega \cup \Gamma$ and $\boldsymbol{\xi} \in \Gamma$; ε is the free term coefficient ($\varepsilon = 1$ if $\mathbf{x} \in \Omega$, $\varepsilon = a/2\pi$ if $\mathbf{x} \in \Gamma$ and $\varepsilon = 0$ if $\mathbf{x} \notin \Omega \cup \Gamma$; a is the interior angle between the tangents of boundary at point $\mathbf{x}$; $\varepsilon = 1/2$ for points where the boundary is smooth, Fig. 2a). Eqs (10) are solved numerically using the BEM. The boundary integrals are approximated using N constant boundary elements, whereas the domain integrals are approximated using linear triangular elements. The domain discretization is performed automatically using the Delaunay triangulation. Since the fictitious source is not defined on the boundary, the nodal points of the triangles adjacent to the boundary are placed on their sides (Fig. 2b).

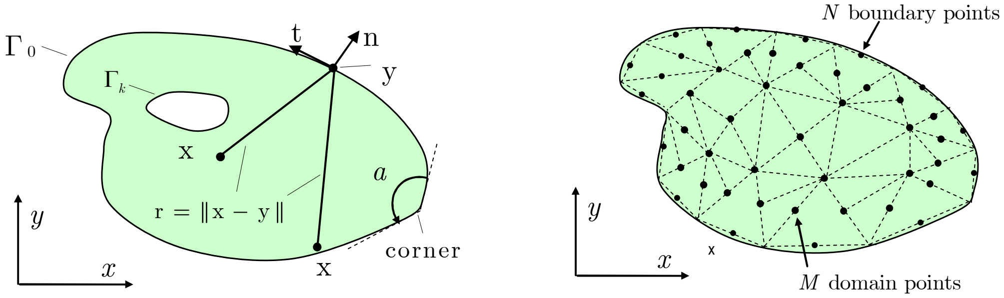


Figure 2. (a) BEM notation and (b) boundary and domain discretization

Thus, after discretization and application of Eqs (10a-c) at the N boundary nodal points we obtain

$$\mathbf{H}\begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\phi}_x \\ \boldsymbol{\phi}_y \end{Bmatrix} + \mathbf{A}\begin{Bmatrix} \mathbf{b_1} \\ \mathbf{b_2} \\ \mathbf{b_3} \end{Bmatrix} = \mathbf{G}\begin{Bmatrix} \mathbf{w_{,n}} \\ \boldsymbol{\phi}_{\mathbf{x,n}} \\ \boldsymbol{\phi}_{\mathbf{y,n}} \end{Bmatrix} \tag{11}$$

where $\mathbf{H},\mathbf{G}$ are $3N \times 3N$ known matrices originating from the integration of the kernel functions on the boundary elements and $\mathbf{A}$ is an $3N \times 3M$ coefficient matrix originating from the integration of the kernel function on the domain elements; $\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ are vectors with the values of the fictitious loads at the M domain points. Applying the boundary conditions Eqs (6)at the N boundary nodal points and using Eqs (7d,e) we obtain

$$\bar{\mathbf{H}}\begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\phi}_x \\ \boldsymbol{\phi}_y \end{Bmatrix} + \bar{\mathbf{G}}\begin{Bmatrix} \mathbf{w_{,n}} \\ \boldsymbol{\phi}_{\mathbf{x,n}} \\ \boldsymbol{\phi}_{\mathbf{y,n}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{\alpha}_3 \\ \boldsymbol{\beta}_3 \\ \boldsymbol{\gamma}_3 \end{Bmatrix} \tag{12}$$

The tangential derivatives of $\phi_{x,t}, \phi_{y,t}$, which appear in the boundary conditions, are expressed in terms of ϕ_x, ϕ_y using a finite difference scheme. Eqs. (11) and (12) constitute a system of $6N$ algebraic equations which can be solved in terms of the boundary quantities $w, \boldsymbol{\phi}_x, \boldsymbol{\phi}_y, w_{,n}, \boldsymbol{\phi}_{x,n}, \boldsymbol{\phi}_{y,n}$. Substituting the boundary quantities in the discretized counterpart of Eq. (10) we obtain the displacements $w, \boldsymbol{\phi}_x, \boldsymbol{\phi}_y$ and their derivatives at the M domain nodal points in terms of the fictitious loads

$$\mathbf{w} = \mathbf{D}^1_{,pq}\mathbf{b}_1 + \mathbf{D}^2_{,pq}\mathbf{b}_2 + \mathbf{D}^3_{,pq}\mathbf{b}_3 + \mathbf{d}_{,pq} \tag{13a}$$

$$\phi_x = \mathbf{S}^1_{,pq}\mathbf{b}_1 + \mathbf{S}^2_{,pq}\mathbf{b}_2 + \mathbf{S}^3_{,pq}\mathbf{b}_3 + \mathbf{s}_{,pq} \tag{13b}$$

$$\phi_y = \mathbf{V}^1_{,pq}\mathbf{b}_1 + \mathbf{V}^2_{,pq}\mathbf{b}_2 + \mathbf{V}^3_{,pq}\mathbf{b}_3 + \mathbf{v}_{,pq} \tag{13c}$$

where $\mathbf{D}^i_{,pq}, \mathbf{S}^i_{,pq}, \mathbf{V}^i_{,pq}$ $(i = 1,2,3)$ are $M \times M$ known matrices and $\mathbf{d}^i_{,pq}, \mathbf{s}^i_{,pq}, \mathbf{v}^i_{,pq}$ known $M \times 1$ vectors. Finally, collocating the governing equations (8) at the M nodal points and substituting the displacements and their derivatives from Eqs (13), we obtain a linear algebraic system of equations

$$\mathbf{K}\begin{Bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\\ \mathbf{b}_3\end{Bmatrix} = \mathbf{F} \tag{14}$$

where $\mathbf{K}$ is $3M \times 3M$ known matrix and $\mathbf{F}$ is $3M \times 1$ known vector. The solution of Eq. (14) gives the values of the fictitious loads which permits the evaluation of the displacements and the stress resultants at the boundary or the interior of the domain by combining equations (3), (4), (7) and (13).

4 NUMERICAL EXAMPLES

4.1 Simply supported square orthotropic plate

A single-layered square plate $a \times a$ with thickness h is subjected to a transverse uniform load f. The plate is simply supported along the boundary (type I, hard) and it is made of an orthotropic material with parameters $E_x = kE_y$, $E_y = 10^6$, $G_{xy} = 0.6 \times 10^6$, $G_{xz} = 0.6 \times 10^6$, $G_{yz} = 0.5 \times 10^6$, $\nu_{xy} = 0.25$. The problem has been solved using $N = 200$ boundary elements and $M = 253$ domain points which result from 420 linear triangular elements. Table 1 presents the deflection and the bending and twisting moments at the center of the plate for various values of the plate to thickness ratio and the parameter $k = E_x / E_y$. The results are compared with those obtained in [11] using FEM for Mindlin and a higher order plate theory.

$k = \dfrac{E_x}{E_y}$	$\dfrac{a}{h}$	$\dfrac{wE_y}{hf}$	$\dfrac{M_x}{a^2 f}$	$\dfrac{M_y}{a^2 f}$	$\dfrac{M_{xy}}{a^2 f}$
3	5	21.87 21.20 21.27	0.0676 0.0695 0.0706	0.0261 0.0278 0.0278	0.0305 0.0319 0.0307
	2.5	2.10 2.02 2.02	0.0652 0.0628 0.0672	0.0292 0.0319 0.0323	0.0320 0.0280 0.0334
10	5	13.53 13.68 13.59	0.1024 0.1051 0.1054	0.0154 0.0146 0.0170	0.0184 0.0177 0.0182
	2.5	1.73 1.69 1.65	0.0890 0.0867 0.0901	0.0230 0.0252 0.0259	0.0230 0.0243 0.0196
40	5	8.84 8.90 8.68	0.1232 0.1266 0.1261	0.0103 0.0122 0.0123	0.0107 0.0109 0.0101
	2.5	1.54 1.52 1.41	0.1010 0.1001 0.1036	0.0197 0.0216 0.0218	0.0179 0.0194 0.0146

Table 1. Deflection and bending moments in example 1 (upper value : present ; middle value : Mindlin [11]; lower value : higher order [11]).

4.2 Simply supported square laminated plate

A five-layer cross-ply (0/90/0/90/0) square plate $0 < x < a$, $0 < y < a$ with total thickness h is subjected to a transverse sinusoidal load $f = q_0 \sin\dfrac{\pi x}{a}\sin\dfrac{\pi y}{a}$. Each laminate is made of an orthotropic material with parameters $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$ and thickness $h_1 = h_3 = h_5 = h/6$ and $h_2 = h_4 = h/4$. The results were obtained with $N = 200$ boundary elements and three different numbers of the domain points, $M = 325, 477, 677$. Table 2 presents the displacement at the center of the plate and the stresses $\sigma_{xx}(a/2, a/2, h/2)$ and $\sigma_{xy}(a, a/2, h/2)$ for various values of the plate to thickness ratio. The results are compared with those obtained by FEM for Mindlin theory and from 3-dimensional elasticity solution [22]. The results are also compared with those of the classical plate theory.

$\dfrac{a}{h}$			$\dfrac{w(a/2,a/2)E_2h^3}{a^4q_0}\times 10^3$	$\dfrac{\sigma_{xx}(a/2,a/2,h/2)h^2}{a^2q_0}$	$\dfrac{\sigma_{xy}(a,a/2,h/2)h^2}{a^2q_0}$
4	3-D [7]		18.668	0.685	0.0394
	FEM [22]		15.602	0.424	0.0228
	AEM	$M = 325$	15.297	0.424	0.0233
		$M = 477$	15.369	0.428	0.0222
		$M = 677$	15.399	0.429	0.0223
10	3-D [7]		6.830	0.545	0.0246
	FEM [22]		6.201	0.487	0.0214
	AEM	$M = 325$	6.046	0.487	0.0216
		$M = 477$	6.115	0.493	0.0214
		$M = 677$	6.145	0.496	0.0217
20	3-D [7]		4.981	0.539	0.0222
	FEM [22]		4.786	0.512	0.0209
	AEM	$M = 325$	4.458	0.489	0.0198
		$M = 477$	4.580	0.504	0.0214
		$M = 677$	4.643	0.510	0.0206
Classic plate theory			4.350	0.359	0.0213

Table 2. Deflection and stresses in the laminated plate in example 2.

4.3 Triangular laminated plate

The triangular clamped plate of Fig. 3a with thickness $h = 0.8$ is subjected to a uniform transverse load $f = 10$. Two cases of the material are studied : (i) the plate is made of one layer of isotropic material with elastic constants $E = 10^6$, $\nu = 0.25$ and (ii) the plate consists of three-layers (0/90/0), where layers 1 and 3 are made of an orthotropic material with parameters $E_1 = 25E_2$, $G_{12} = G_{13} = 0.5E_2$, $G_{23} = 0.2E_2$, $E_2 = 10^6$, $\nu = 0.25$ and thickness $h_1 = h_2 = 0.05h$, while layer 2 is made of the isotropic material of case (i) with $h_2 = 0.9h$ (Fig. 3b). The results were obtained with $N = 210$ boundary elements and $M = 272$ domain points resulting from 451 linear triangular elements (Fig. 4). Fig. 5-6 present the displacements and the stress resultants at line $y = 1$ for the two cases. Fig. 7-9 present the contours of the displacements and the bending moment M_x and M_y.

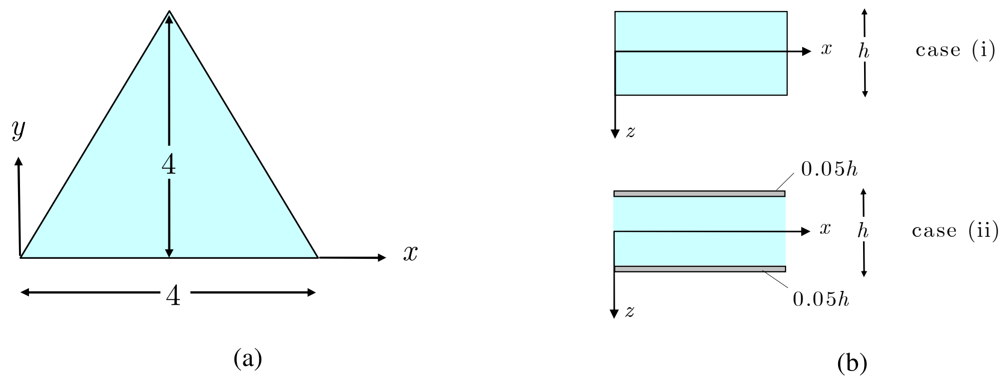


Figure 3. (a) plate geometry and (b) laminates in example 3

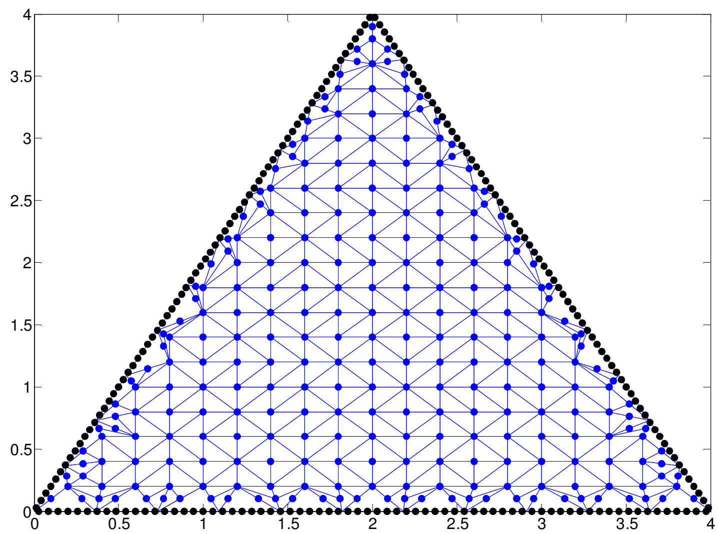


Figure 4. Boundary and domain discretization in example 3

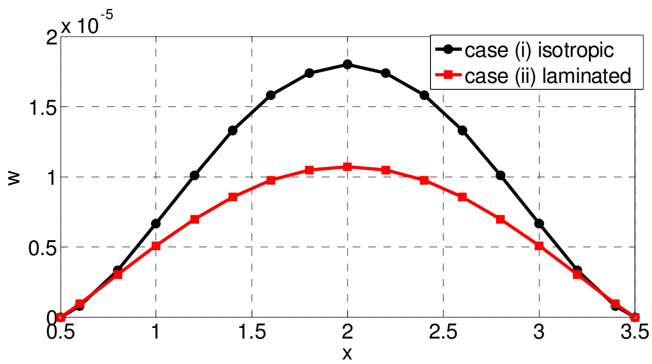
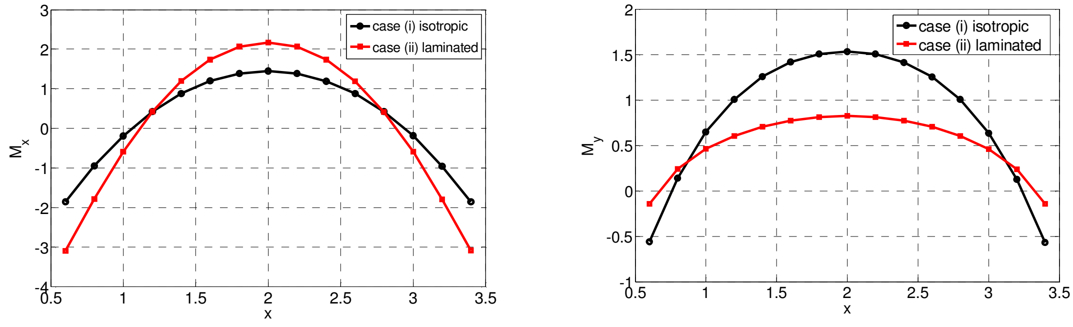
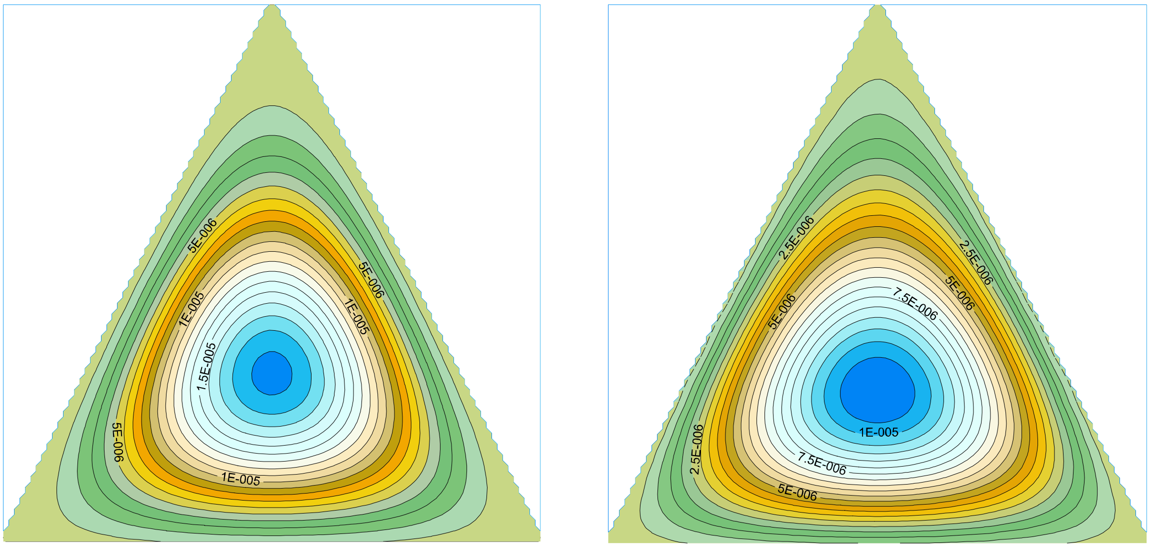


Figure 5. Deflection along the line $y = 1$ in example 3



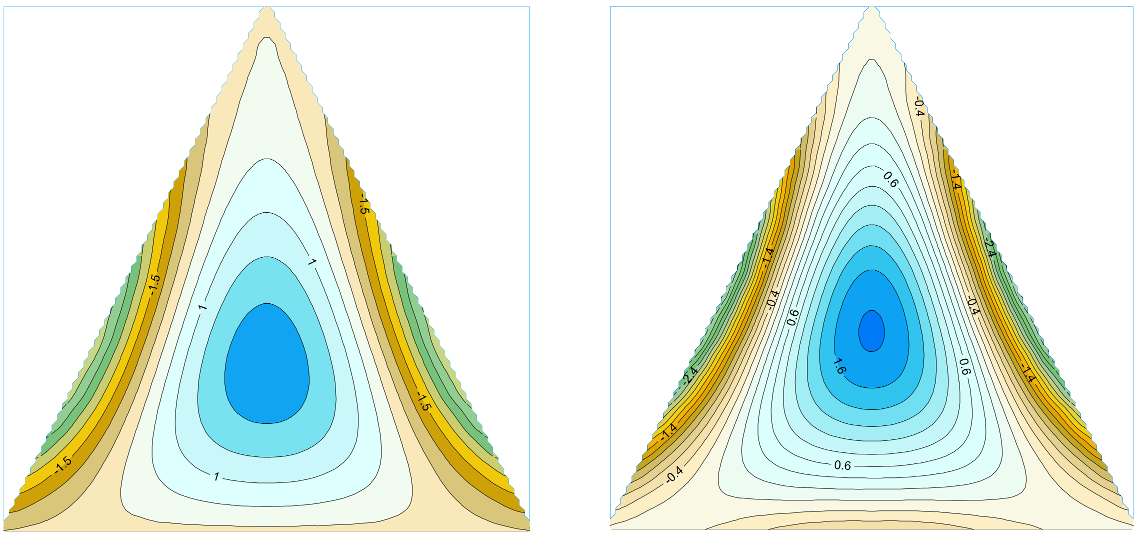
(a) (b)

Figure 6. Bending moments along the line $y = 1$ (a) M_x and (b) M_y in example 3



(a) (b)

Figure 7. Deflection contours for (a) case (i) and (b) case (ii) in example 3



(a) (b)

Figure 8. M_x contours for (a) case (i) and (b) case (ii) in example 3

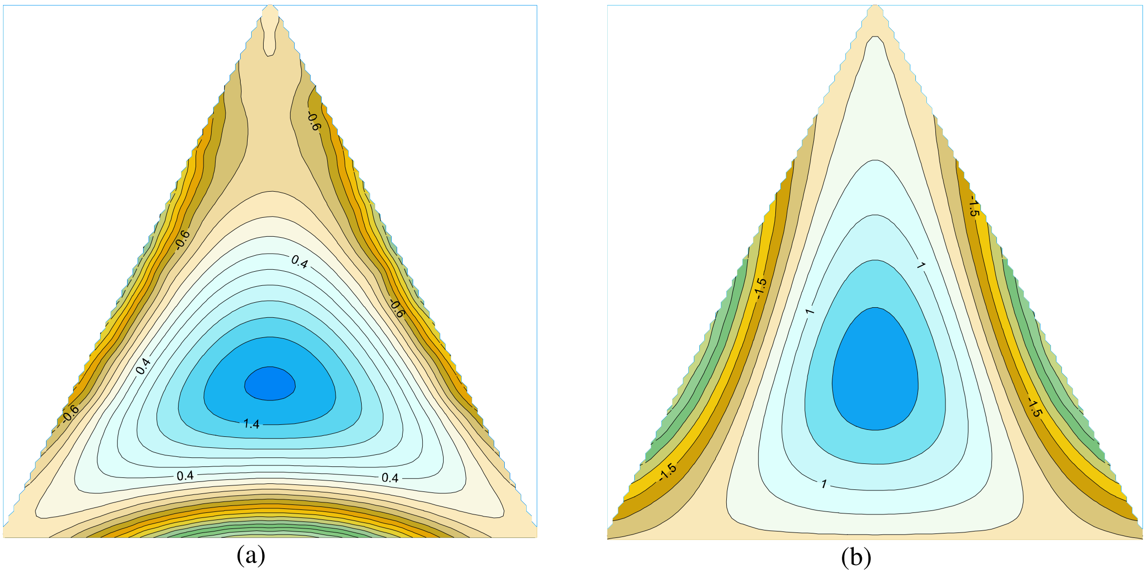


Figure 9. M_y contours for (a) case (i) and (b) case (ii) in example 3

5 CONCLUSIONS

In this paper we study the bending problem of thick layered anisotropic plates using the BEM. The Mindlin plate theory is adopted which gives accurately the global response of moderate thick anisotropic plates. The plate may have arbitrary shape and subjected to domain and boundary loads under any type of admissible boundary conditions. The governing equations are written in terms of the transverse deflection of the middle surface and its rotations about the x and y axis. It is a system of coupled partial differential equations (PDE) of second order, which is solved using the analog equation method. The original equations of the problem are substituted by three uncoupled Poisson's equations under fictitious loads. The Poisson's equations are solved using the BEM with constant boundary elements and linear triangular elements for domain discretization. Several illustrative plate problems are solved. The results are compared with FEM solution of Mindlin plate theory, higher order theories and 3-dimension solutions. From the comparison of the results, we conclude that the presented method gives accurate results for small number of constant boundary elements and linear triangular domain elements. Moreover, Mindlin plate theory gives acceptable results even for smal values of the plate to thickness ratio.

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