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OPTIMISATION OF HIGH STRENGTH STEEL PRESTRESSED TRUSSES

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Abstract. *The present study investigates the optimal structural configuration for steel planar tubular trusses and examines the effect of applying prestress via cables embedded to the bottom chord of the trusses on the response of the structure. At the first stage, topology optimisation of steel planar trusses subjected to vertical loads is performed with the optimisation tool of FE software Abaqus. Various parameters of the optimisation process are examined, leading to two optimised configurations-one with arched and one with straight chords. The two optimised geometries are exported to AutoCAD, where the truss configurations are established. The truss members are designed thereafter according to EN 1993-1-1. The optimised trusses are finally subjected to non-linear static analysis with Riks procedure, thus allowing for the evaluation of the structural response. The effect of varying steel grade and prestress level on the structural response is investigated and the results are presented in normalised load-midspan displacement graphs. Emphasis is given on the results for High Strength Steel (HSS) prestressed trusses, where the combination of the enhanced material properties along with the benefit of prestressing optimises the structural performance. In the last part, the effect of prestressing on the structural response of steel trusses is discussed and relevant conclusions are driven.*

1 INTRODUCTION

Steel trusses –namely triangulated systems of straight interconnected structural steel elements– have long found many applications in building construction. The advantages of steel as a material –the high strength to weight ratio, the predictable material properties and the speed of construction– make steel trusses structures of high performance [1]. Over the past decades higher steel grades have been introduced in the construction sector and have attracted considerable attention from designers and researchers, as their use can potentially lead to smaller member sizes and hence to lighter structures, reduced foundation, transportation and erection costs and hence profound sustainability benefits. However, for large column-free spaces structural applications, the utilisation of high strength steels offers little help with stiffness related issues, which often govern the design. Hence, establishing ways to mitigate excessive deflections will allow the full benefit of high strength steels to be exploited. Prestressed steel trusses which have emerged as a promising structural system suitable for long unsupported spans in Australia several decades ago [2, 3] can also be applied to high strength steel trusses, where storing energy under increasing preload can allow for minimum material consumption, thus maximising the exploitation of the increased material yield strength. The present study aims to enhance the current understanding by investigating the structural response of a novel structural system involving prestressing of cables embedded to the bottom chord of tubular trusses.

2 BACKGROUND

2.1 Optimisation

Optimisation techniques are used to find feasible solutions to real-life problems, from mathematical programming to operations research, economics, management science, business, medicine, life sciences, artificial intelligence [4]. Optimisation concerns a branch of mathematics aiming to obtain the conditions that give the extreme value of a function, or many functions, subjected to certain constraints with respect to variables and can be generally described as follows:

Find $X = (x_1, x_2, ... x_n,)$ that minimises (or maximises) $f_i\ (X), i = 1,2, ... n_0$, subject to:

$g_j(X) \leq 0, \qquad j = 1,2,\dots,n_g$

$h_k(X) = 0, \qquad k = 1,2,\dots,n_e$

$x_m{}^l \leq x_m \leq x_m{}^u, \qquad m = 1,2,\dots,n_s$

where X is the vector of n design variables, $f_i\ (X)$ is an objective or merit function and $g_j(X)$ and $h_k(X)$ are the inequality and equality restraints respectively [4]. When the design objectives used are structural criteria that evaluate the merit of a design such as minimum construction cost, minimum life-cycle cost [5, 6], minimum weight, and maximum stiffness [7], then the optimisation is defined as structural optimisation. In other words, structural optimisation is an iterative process that aims to produce lightweight but also rigid and durable structural components and can be divided into three categories: shape, size and topology optimisation.

Topology optimisation techniques utilise advanced mathematical tools in order to produce optimised structural landscapes and have already been applied in many areas, like in architectural design in order to optimise the number of openings within a structure, in the construction of high-rise buildings, [8] as well as in the design of compliant mechanisms [9]. Given an initial material distribution, topology optimisation produces a new landscape, by scaling the relative densities of the elements in the design domain, accounting for an improved weight-to-stiffness ratio. Topology optimisation is utilised in the first step of the herein presented study in order to find optimal truss landscapes.

2.2 HSS prestressed trusses

HSS was relatively recently introduced as a structural material and certain projects have started applying HSS, exploiting its enhanced material properties [10]. In order to design HSS structures, it is necessary to know the structural response under various load configurations. Towards this direction, much of the published research has focused on the compressive, flexural and cross-sectional response of HSS structural members and components [11, 12, 13]. A new part of Eurocode for the design of HSS has also been recently published [14]. However, the focus of most research has been on the strength of HSS structures, with little emphasis placed on their Serviceability Limit State (SLS), which may govern their design. By applying prestress prior to the application of the live loads, a favourable deformation pattern, which counteracts the effects of gravity and snow loading, emerges, thus overcoming the problem with deflection limits. The concept of prestressing steel structures firstly made its appearance some decades ago [15]. In steel trusses, prestress has been used as an erection method of arched frames [2, 3] – this technique of erecting prestressed arched trusses with sliding joints through the tensioning of a cable on their bottom chord was also recently applied for the construction of temporary steel structures in Hyde Park, as can be seen in figure 1. To further investigate the response of these structures, the tensile performance of prestressed elements has been experimentally and numerically investigated [16].

Figure 1. Arched prestressed trusses in Hyde Park, London

3 OPTIMISATION PROCESS

The latest versions (6.11 and on) of the general purpose finite element software Abaqus include Abaqus Topology Optimisation Module (ATOM), a useful optimisation tool that allows for topology or shape optimisation. The topology optimisation removes volume to find more efficient topologies, whereas the shape optimisation moves nodes to smooth peak stresses. To achieve this, Abaqus uses a gradient based optimisation technique. The design variables are set as the relative densities and are continuous, whereas the intermediate density elements are penalised. The topology optimisation process can be seen in figure 2 [17]:

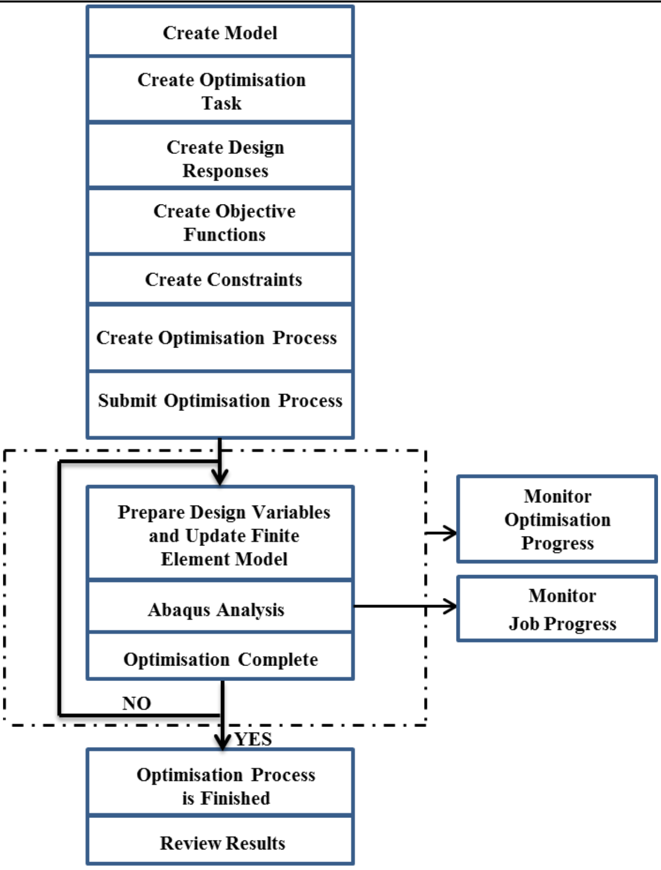

Figure 2. Optimisation Process in Abaqus

In the optimisation process shown in figure 2, the design responses provide variables for the optimisation solver, the objective functions define how those design responses will be used (sum/min/max/formula), whilst the constraints determine bounds for the optimisation solver. The present study utilises ATOM to conduct topology optimisation and establish optimal truss geometries.

3.1 Development of the model

The overall geometry of the trusses to be investigated (i.e. span to depth ratio, straight or curved chords) is based on commonly employed structural configurations and is discretised with 2D solid elements CPS4R. CPS4R are two-dimensional quadrilateral linear reduced-integration plane stress elements and are recommended for modelling continuum elements in the 2D space. The boundary conditions applied simulate a simply supported beam. The optimisation process is executed under linear static analysis and hence only the elastic material properties (i.e. Young modulus and Poisson's ratio) of steel are introduced. The chosen span-to-depth ratio at midspan and the chosen length, as depicted in figure 3, were based on an initial study to establish the geometry of a truss that will be subsequently tested at Imperial College, London, within HILONG Project [RFSR CT 2012-00028], as shown in figure 4.

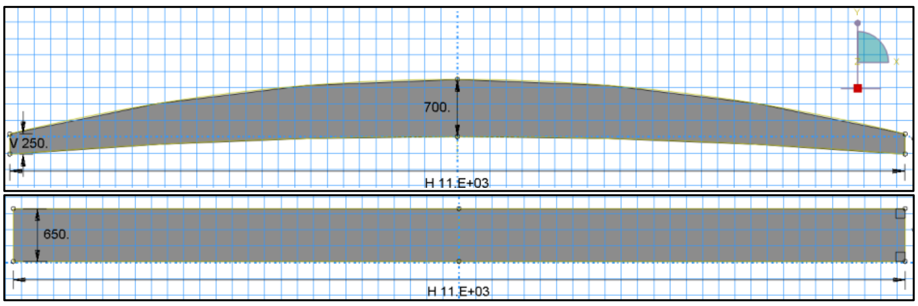

Figure 3. 2D models to be optimised

Figure 4. HSS trusses to be tested at Imperial College

According to the optimisation process of figure 2, after the development of the model, the objective function and the constraints need to be specified. In order to find the optimal truss topologies, the design objective to be maximised under certain constraints is the stiffness. Maximising the stiffness is equivalent to minimising the strain energy (i.e. the energy stored to a body due to deformation) which is therefore set as the objective function of the optimisation process. The minimisation of the strain energy is decided to be subject to a volume constraint, which is introduced through the ratio $V_{final}/V_{initial}$, where V_{final} and $V_{initial}$ are the final and initial volume of the modelled geometry respectively.

3.2 Definition of initial parameters

As for all the FE analyses, a mesh convergence study is performed in the first step. As can be seen in figure 5, a coarse model gives insufficient results, whilst the reduction of the element size to half leads to good results within reasonable computational time.

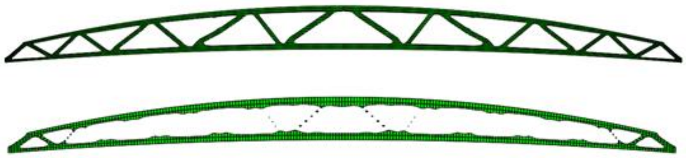

Figure 5. Last design cycle of optimisation process for fine and coarse mesh respectively

Moreover, the convergence parameter of the optimisation process was increased from the default value 0.0001 to 0.00001, thus allowing more iterations and hence more accurate results. For the selected element size and convergence parameter, different load configurations have been studied, including uniformly distributed loads on the top chord and concentrated loads at specific points. A single concentrated load was applied at the midpoint of the top chord. This configuration seems to produce good results within reasonable time and was therefore applied throughout this study.

3.3 Definition of the volume constraint

One important step of the optimisation process is the definition of the constraint, being the ratio $V_{final}/V_{initial}$. Different values of this ratio are examined in this step. Figure 6 shows the results of the optimisation process by varying the ratio of the volume constraint from 0.3 to 0.5 and to 0.7, whilst keeping the rest remaining parameters constant. As can be observed, the ratio $V_{final}/V_{initial}$=0.3 gives the best results and is therefore adopted for the optimisation process of the present study.

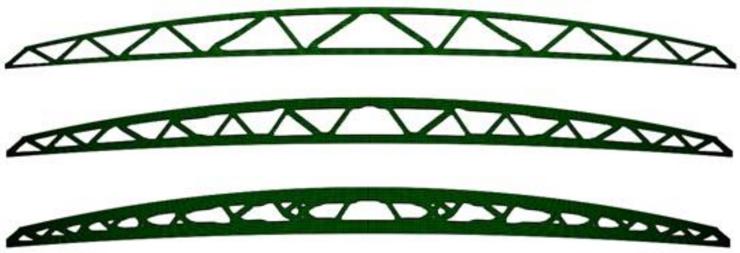

Figure 6. Influence of the volume constraint - $V_{final}/V_{initial}$ 0.3, 0.5, 0.7 respectively

3.4 Influence of the ratio H/L

During the study of the topology optimisation with Abaqus, it has also been found that the height-to-span ratio H/L of the initial geometry is linked with the final optimal configuration, as can be observed in figure 7, where the number of diagonals of the optimised geometries decreases, as the initial ratio H/L increases.

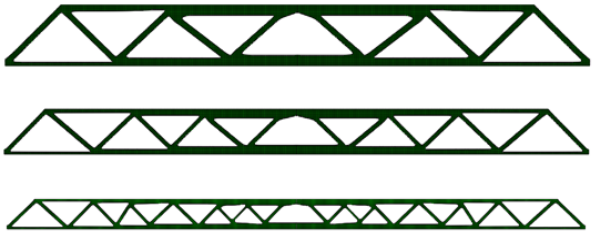

Figure 7. Optimised trusses for H/L=0.10, 0.075, 0.05 respectively

3.5 Modelling with stringers

Having established the basic parameters of the optimisation process, a new 2D model with stringers on the bottom and top chord is created. Stringers are beam elements, assigned along any edge of a 2D planar part and bonded to the edge with the same nodes and topology as the underlying entities of the part [17]. As can be seen in figure 8, by assigning appropriate beam sections to the stringers, only the topology of the inner part is optimised, giving the optimal layout of the truss diagonals. For the optimised trusses of figure 8, the objective function (strain energy at design cycle i/initial strain energy) and the constraint (volume at design cycle i/initial volume) as a function of the optimisation design cycle are presented in figure 9. For both trusses, the strain energy minimises, whilst the ratio $V_{final}/V_{initial}$ tends to 0.3, pursuing the most optimal truss topology.

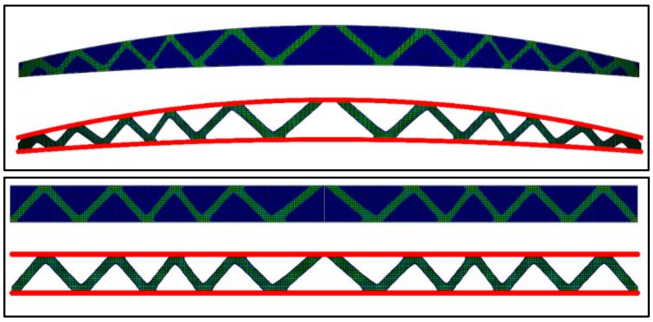

Figure 8. Optimised truss topologies with stingers on the top and bottom chord

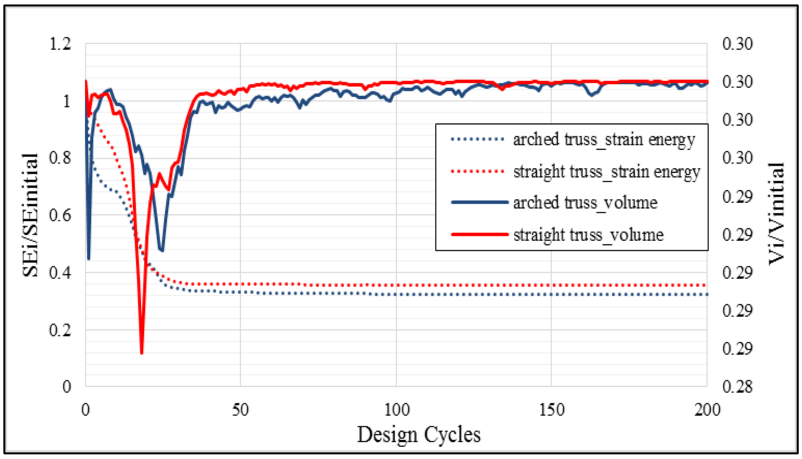

Figure 9. Objective function and volume constraint vs optimisation design cycle

4 DESIGN OF THE OPTIMISED TRUSSES

Having successfully computed the two optimal truss geometries, the landscapes of the last optimisation cycle are exported from Abaqus and imported to AutoCAD, where the final truss configuration is established by connecting the midlines of the diagonal areas.
The truss members are thereafter designed in accordance with Eurocode [18] and based on the design examples given in [19]. The permanent and variable actions as well as the partial factors used for the design are presented in Table 1.

Permanent Actions		Ultimate Limit State (ULS)	
Self weight of roof construction	0.75kN/m^2	Partial factors for actions	
Self weight of services	0.15kN/m^2	Partial factor for permanent actions	$\gamma_G = 1.35$
Total permanent actions	**0.90kN/m^2**	Partial factor for variable actions	$\gamma_Q = 1.5$
Variable Actions		Reduction factor	$\xi = 0.925$
Imposed roof load	0.60kN/m^2	**Serviceability limit state (SLS)**	
Total variable actions	**0.60kN/m^2**	Partial factors for actions	
Assumption: purlins every 6m		Partial factor for variable actions	$\gamma_Q = 1.0$

Table 1. Permanent and variable actions - Partial factors for ULS and SLS

Based on Table 1, the two trusses under vertical loading are statically analysed and a uniform section is chosen for the bottom chord, the diagonals and the top chord respectively. The minimum structural steel grade S235 is considered for the design, leading to the selection of square hollow section (SHS) 60x60x7 for the bottom chord, SHS 40x40x4 for the diagonals and SHS 80x80x7 for the top chord of both trusses. The midspan deflection of the trusses for the SLS load combination is also checked for the selected sections.

Design of members		Area (mm^2)	I (mm^4)	steel grade
Bottom Chord	60x60x7	1399.9	639330.1178	S235
Top Chord	80x80x7	1679.9	1087804.522	S235
Diagonals	40x40x4	548.5	115984.1556	S235

Table 2. Design of the truss members

5 NON-LINEAR RESPONSE OF PRESTRESSED STEEL TRUSSES

In order to evaluate the benefit provided to steel trusses by the embedding of a prestressed cable to the bottom chord, five different steel grades (S235, S275, S355, S460 and S690) for the truss members and four prestress levels are examined for two trusses, leading to a total of 40 non-linear static analyses. The four prestress levels correspond to the following:

1) No cable
2) Cable embedded to the bottom chord without prestress
3) Cable embedded to the bottom chord and 0.5 P_{opt} prestress
4) Cable embedded to the bottom chord and P_{opt} prestress

where P_{opt} is the optimal prestress level, for which both the tube and the cable yield simultaneously under tensile loading and can be calculated by the following formula [16]:

$$P_{opt,t} = \left(\frac{A_c + A_t}{A_t E_t + A_c E_c}\right) * \left(f_{cy} E_t - f_{ty} E_c\right) \quad (1)$$

where A is the cross-sectional area, E is the Young’s modulus, f_y is the yield stress and the subscripts t and c stand for tube and cable respectively. For all the cases, a cable of A=150 mm^2 and f_y=1860MPa is considered.

5.1 Modelling of the truss

The two optimal truss configurations, as established previously, are now imported as 2D planar models to Abaqus. For the truss members, 2 noded linear beam elements in plane (B21) are used, whereas the cable is modelled with 2 noded linear truss elements. For the material properties of all the steel grades, a linear elastic-perfectly plastic response is considered. Appropriate constraints are introduced in order to tie the degrees of freedom of the bottom chord elements with those of the truss elements simulating the cable. The model is loaded at five points -corresponding to the points of purlins- on the top chord. Prestress is introduced at a first step as initial stresses in the cable. Non-linear static analysis with the modified Riks procedure [20] is then carried out in order to evaluate the structural response of the whole structure. Riks analysis, which is a variation of the classic arc-length method, allows for both geometric and material non-linearities to be properly considered and the post ultimate path to be traced. Initial geometric imperfections of the truss are also considered and are assumed to have the shape corresponding to the lowest elastic buckling mode shape, which was found by means of a linear eigenvalue buckling analysis at an earlier step. The assumed imperfection amplitude was taken as L/1000, where L was the buckling length of the component undergoing buckling in the linear buckling analysis.

5.2 Results

The total applied vertical load is normalised by the squash load (A*fy) of the bottom chord and plotted against the midspan deflection of the truss, allowing the evaluation of the structural response. Figure 10 shows typical deformed shapes of the two truss geometries considered. Figure 11 presents the response of the optimised S460 trusses, leading to the following conclusions for the four prestress levels:

1) Based on the assumption of elastic-perfectly plastic material, the trusses without cable in the bottom chord behave linearly up to the yielding point, where a yield plateau is formed.
2) The embedding of a cable to the bottom chord makes the whole structure more stiff (increased inclination of the linear part), whilst the maximum load is increased as well.
3) By prestressing to $0.5P_{opt}$ level the cable of the bottom chord, the maximum load becomes even higher and the corresponding deflection reduces
4) The highest load and the lowest midspan displacement is achieved by prestressing the cable to P_{opt} level

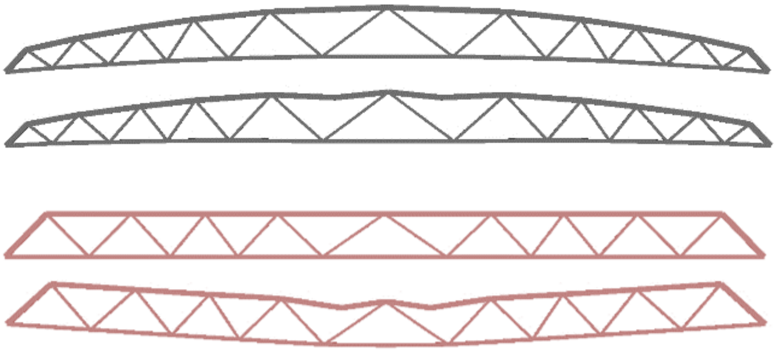

Figure 10. Typical deformed shapes for the two truss configurations

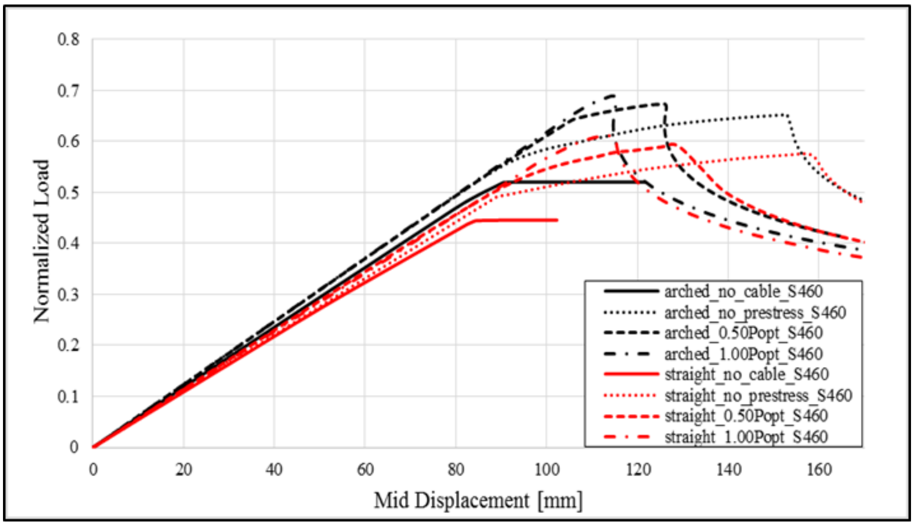

Figure 11. Normalised load vs midspan displacement for S460 trusses

6 DISCUSSION

In order to evaluate the effect of the prestress on the structural response of steel trusses, Figure 12, 13 and 14 present the normalised load-midspan displacement graphs for the arched truss, considering different steel grades for the truss members.

Figure 12 shows that by embedding a cable to the bottom chord of a steel truss and prestressing it to P_{opt}, the ultimate load increases significantly (with an average value of 35% for the different steel grades).

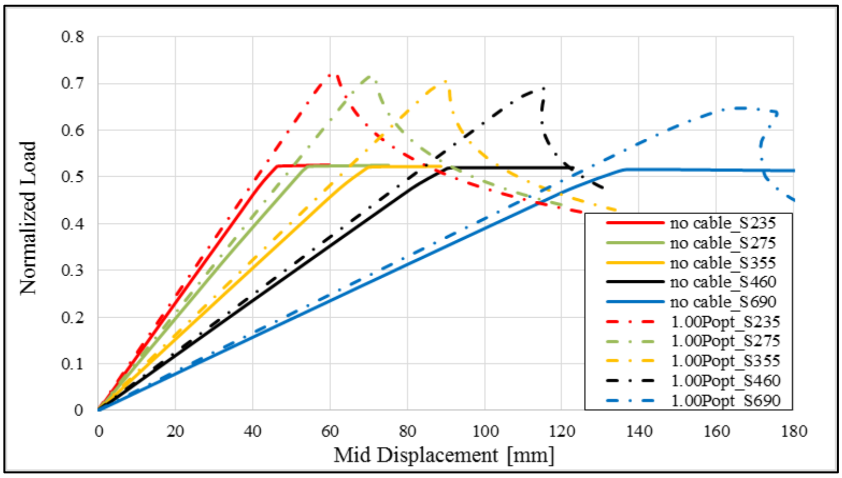

Figure 12. Normalised load vs midspan displacement for varying steel grade

Figure 13 presents the results for zero and P_{opt} prestress level and reveals that for all cases, the midspan deflection of the truss at the ultimate load decreases significantly, when P_{opt} prestress is assigned at the cable embedded on the bottom chord. The reduction in midspan displacement was found to have an average value of 28.5% for the different steel grades.

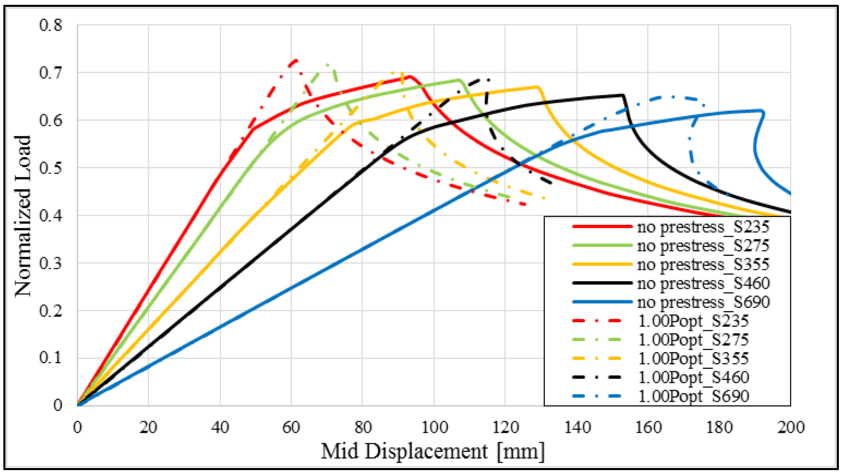

Figure 13. Normalised load vs midspan displacement for zero and $1.00P_{opt}$ prestress

Similar to these graphs are the results for the truss with straight bottom and top chord. It is therefore concluded that for the studied truss configurations, the embedding of a prestressed cable to the bottom chord can positively contribute to a significant increase of the stiffness and capacity of the structure, which in case of HSS planar trusses could allow for a better exploitation of the material properties.

7 CONCLUSIONS

The paper has investigated the response of prestressed steel trusses. As a first step, optimal truss topologies were studied with the use of ATOM tool. After establishing two optimised truss landscapes, the truss members were designed according to European standards. Their response for varying prestress level and steel grade was then studied with the execution of non-linear static analysis. The results were plotted in normalised-load-midspan displacement curves, allowing the investigation of the optimal magnitude of prestress level with respect to the steel grade. It was shown that for all cases, the introduction of prestressing to the bottom chord increases the ultimate load and decreases significantly the corresponding midspan deflection. Further research is underway to investigate the benefits of prestressing for trusses with various span-to-depth ratios and member sizes for the top and bottom chord. This benefit seems to be of particular interest for higher steel grades, as it can facilitate the design of iconic structures covering long spans and thereby push further the limits of what is considered achievable in modern structural engineering practice.

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