ON THE NONLINEAR CYCLIC BEHAVIOR OF CIRCULAR CONCRETE-FILLED STEEL TUBES

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Abstract. This paper presents a simple and efficient analytical model for the cyclic behavior and strength capacity of circular concrete-filled steel tubes (CFT) under axial load and cyclically varying flexural loading. Firstly, an accurate nonlinear finite element model is created using the ATENA software. The validity of this model is established by comparing its results with those of experimental data published in the literature. Then, using this finite element model, an extensive parametric study is conducted to create a databank of hysteretic behavior of circular CFTs, involving numerous circular CFT columns with different diameter to thickness ratios, steel tube yield stress and concrete strength. On the basis of this computational study, expressions are developed to determine the necessary phenomenological parameters of the well-known Ramberg-Osgood hysteretic model. Additionally, a proposed method is extended involving analytical relations for the capacity of circular CFT columns which provide a direct and efficient representation of the ultimate strength of circular CFT columns. Comparisons between analytical and experimental results demonstrate that the proposed analytical model provides good convergence for the cyclic behavior of circular CFT columns.

1. INTRODUCTION

Steel members are characterized by high tensile strength and ductility, while concrete members have the advantages of high compressive strength and stiffness. Composite members combining steel and concrete result in members enjoying the advantageous qualities of both materials, i.e., sufficient strength, ductility and stiffness [1]. Concrete-filled steel tube (CFT) columns are widely used in heavy constructions because they provide excellent static and earthquake-resistant properties, such as high strength, high ductility, high stiffness, and large energy-absorption capacity. CFT columns provide benefits obtained both from steel and concrete: a steel tube surrounding a concrete column not only assists in carrying axial load but also confines the concrete. Furthermore, it eliminates the permanent formwork, which reduces construction time and cost, while the concrete core takes the axial load and prevents or delays local buckling of the steel tube. However, they are scarcely adopted in the construction industry, mainly due to the lack of understanding of their structural behavior and reliable design recommendations [1, 2].

Moment resisting frames (MRFs) composed of CFT columns combined with steel beams are one form of composite construction. The combination between CFT-MRF provides a ductile and lightweight frame with the added stiffness of composite columns to control lateral drift [3]. Although, the research on CFT columns has been ongoing worldwide for decades and significant contributions have been made by many researchers, some cyclic loading experiments (i.e. Inai et al., [4]; Varma et al., [5]) have been conducted in order to examine their hysteretic behavior. There are many types of CFT columns, as illustrated in Figure 1. This study is focused on the circular CFT columns, which outmatch against square columns such as: their moment enhancement ratios are greater due to the larger confinement of the concrete core, their circular steel tubes have advantage of restraining local buckling limiting the deterioration phenomena, their flexural strength and ductility are higher [3,4].
The purpose of this paper is to propose an accurate analytical model to simulate the cyclic behavior of circular CFT column. This model is based on concentrated plasticity theory, it is simple and it can be used in seismic analysis of composite MRFs in combination with Ruaumoko program for simulating accurately the complex behavior of a CFT member under axial force and bending moment. The main objective is the determination of the Ramberg-Osgood hysteretic model parameters which are available in Ruaumoko [6] or other similar nonlinear structural analysis programs. These parameters are defined empirically on the basis of an extensive response database created with the aid of a refined CFT finite element model that involves both concrete and steel nonlinear behavior, using ATENA program [7] and following the methodology developed by Skalomenos et al. [3] who proposed hysteretic models for simulating the cyclic response of square CFT columns. Based on the findings of their study and those obtained here, comparisons are also made between the cyclic responses of circular and square CFT columns.

2. FINITE ELEMENT MODEL STRUCTURE FOR CIRCULAR CFT COLUMNS

To study the actual behavior of circular CFT column, three-dimensional non-linear finite element models were constructed using the finite element software ATENA program. According to the experimental procedures of Inai et al. [4], a circular CFT column with length 1.5 m was structured which was fixed at its base. At its top a constant axial load (P) and a lateral loading (H) were subjected as shown in Figure 2. At the top of surface of the column the axial load (P) was applied via a rigid plate and a pre-stressed cable aiming at indicating the base of the column and in the direction of the chord of its displaced shape. Due to symmetry, only a half of the column is analyzed. The nodes on the symmetry surface were restricted on Y direction. For solving the nonlinear equations of motion Newton Raphson method was used in ATENA.

In the finite element mesh, both the concrete core and the steel tube are modeled by 20-node shell elements and 8-node solid elements, respectively. In the modeling of steel tubes, parameters such as the nonlinear behavior of confined concrete, the cyclic local buckling of steel tubes and the interface between concrete and steel tube are taken into account. Another important criterion for the modeling process is the choice of the element type and mesh size that provide accurate results with reasonable computational time [3].

3. MODELING OF CIRCULAR CFT COLUMN

3.1 Confined concrete modeling

Since the confining effect causes the concrete core to behave in a triaxial compressive stresses due to interaction between steel tube and concrete in a CFT column, the failure of concrete is dominated by the
compressive failure surface expanding with increasing hydrostatic pressure. Hence, a suitable model that describes the triaxial strength of concrete in terms of three independent stress invariants \((\xi, \rho, \theta)\) is the hardening/softening plasticity model, which is based on Menétrey and Willam [8] failure surface. This model can be used to simulate the concrete cracking, the crushing under high confinement and the crack closure due to the crushing in other material directions. The failure surface \(F_{3p}^p\) of Menétrey and Willam is defined as:

\[
F_{3p}^p = \left[\sqrt{1.5} \frac{\theta}{f_c} \right]^2 + m \left[ \frac{\rho}{\sqrt{6f_c}} r_1(\theta, \epsilon) + \frac{\epsilon}{\sqrt{3f_c}} \right] - c = 0
\]  

where

\[
m = \frac{3f_c^2 - f_t^2}{f_t^2} \frac{e}{e + 1}, \quad r_1(\theta, \epsilon) = \frac{4}{2(1 - e^2)^2} \cos^2 \theta + (2e - 1)^2
\]

In the above equations, \((\xi, \rho, \theta)\) are Heigh-Westergaard coordinates, \(r_1(\theta, \epsilon)\) is the elliptic function, \(m\) is the friction parameter, \(f_c\) and \(f_t\) denote the uniaxial compressive strength and the uniaxial tensile strength, respectively. The eccentricity parameter \(e\) is ranged from 0.5 to 1.0 and describes the roundness of the failure surface. The failure surface has sharp corners if \(e = 0.5\) and is fully circular around the hydrostatic axis if \(e = 1.0\). In this study, since the predicted model has circular shape, the eccentricity \(e\) is equal to 1.0. Finally, the cohesion parameter is ranged from 0 to 1.0 and it controls the hardening/softening for Menétrey-Willam surface.

### 3.2 Steel tube modeling

The hysteretic behavior of thin-walled steel tubes is strongly affected by the local buckling with cyclic metal plasticity. For this purpose in order to express the local buckling of steel tubes we incorporate Von-Mises plasticity theory in conjunction with the Armstrong and Frederic nonlinear kinematic hardening rule. The ability of the proposed model describes accurately the Bauschinger effect. Von-Mises plasticity model, also called \(J_2\) plasticity, is based only on one parameter \(k\). The yield function is the following:

\[
F^p(\sigma_p) = \sqrt{J_2} - k(e_p^p) = 0, \quad k = (e_p^p) = \sqrt{\frac{1}{3} \sigma_p(e_p^p)}
\]  

where \(J_2\) denotes the second invariant of stress deviator tensor, the parameter \(k\) is the maximal shear stress and \(\sigma_p\) is the uniaxial yield stress, which controls the isotropic hardening of the yield criterion and it is defined as:

\[
\sigma_p = (e_p^p) = \sigma_p + H e^p_{eq}, \quad e^p_{eq} = \sum_{i=1}^{ninc} 2/3(\Delta e^p_{i}; \Delta e^p)
\]

\(H\) is the hardening modulus and \(e^p_{eq}\) the equivalent plastic strain calculated as a summation of equivalent plastic strains during the loading history. In case, Von-Mises model could be used to model cyclic steel behavior including effect, the yield function can be expressed as:

\[
F = \sqrt{1/2(\sigma^\prime - X)(\sigma^\prime - X)} - k(e^p_{eq}) - (r_1 - 1)k_0 = 0
\]  

where \(\sigma^\prime\) is the deviatoric stress, \(k_0\) is the initial value of \(k(e_p^p)\) according to Eq. (5) and \(X\) is the so called back stress controlling the kinematic hardening:

\[
\Delta X = 2/3k_1 \Delta e^p - k_2 X \Delta e^p_{eq}
\]

Eq. (6) has the following solutions:

\[
X = \begin{cases} \frac{2k_1}{3k_2} + (X_0 - \frac{2k_1}{3k_2}) \exp[-k_2(e^p_{eq} - e^p_{eq0})], & \Delta e^p_{eq} \geq 0 \\ -\frac{2k_1}{3k_2} + (X_0 + \frac{2k_1}{3k_2}) \exp[-k_2(e^p_{eq} - e^p_{eq0})], & \Delta e^p_{eq} < 0 \end{cases}
\]

where the quantities \(r_1, k_1\) and \(k_2\) are the material parameters for the cycling response. If \(r_1 \neq 0\), the cyclic model is activated and it controls the radius of Von-Mises surface. If \(r_1 = 1\), the yielding will start exactly when \(\sigma_p\) is reached. For lower values the non-linear behavior starts earlier and slope of the response is mainly affected by parameter \(k_1\) (large value-higher slope). Parameter \(k_2\) affects the memory of the cyclic response. In this study, based on our preliminary experimental results, we concluded that the appropriate values for \(r_1, k_1\) and \(k_2\) were...
found to be $0.40$, $50 \times 10^3$ MPa and 90, respectively. Moreover, the Poisson’s ratio $\nu_s$ and the elastic modulus $E_a$ are assumed to be 0.3 and 210 GPa, respectively.

### 3.3 Interface modeling between steel and confined concrete

Using an accurate model, which illustrates the actual contact behavior, a simulation of the interface action between steel tubular column and confined concrete is necessity. This interaction is modeled by a special 8-node interface element, called gap element, which is available in ATENA. When these two surfaces come into contact, contact pressure acts on a representative surfaces and frictional stress occurs in the direction tangential to the contact surface. This kind of behavior is based on Mohr-Coulomb criterion with cut-off. The constitutive relation for a general three-dimensional case in given in terms of tractions on interface planes and relative sliding and opening displacements and it is mentioned as:

$$
\begin{pmatrix}
\tau_1 \\
\tau_2 \\
\sigma
\end{pmatrix} =
\begin{bmatrix}
K_{nt} & 0 & 0 \\
0 & K_{nt} & 0 \\
0 & 0 & K_{nn}
\end{bmatrix}
\begin{pmatrix}
\Delta \nu_1 \\
\Delta \nu_2 \\
\Delta u
\end{pmatrix}
$$

where $\tau$ is the shear stress, $\sigma$ the normal stress, $\Delta \nu$ and $\Delta u$ are the relevant sliding and opening displacement respectively. $K_{nn}$ and $K_{nt}$ denote the initial elastic normal and shear stiffness respectively and they are assumed to be equal to $10^5$ MPa [9]. Additionally, it is assumed that the contact surfaces are not allowed to penetrate each other, the friction between the two faces with a coefficient equal to 0.4 is maintained as long as the surfaces remain in contact and no tension strength exists between the two faces, thereby allowing the contact surfaces to separate.

The accuracy of ATENA finite element model is validated comparing the two cyclic loads with constant axial load experiments conducted by Inai et al. [4]. Figure 3 illustrates the curves of moment ($M$) versus rotation angle ($\theta$) of finite element analysis are plotted compared with the experimental data which reveal similar performance in both cases.

![Figure 3. Moment–Rotation Angle response of test specimens SC4A4C and SC6A9C [4] compared with finite element analysis of ATENA program.](image)

Subsequently, the basic stages, which presented in Figure 3, are described analytically [10]:

**Stage 1:** Elastic stage (from the center of axes to point A). During this stage, steel and concrete bear load independently. The yielding of steel occurs at point A.

**Stage 2:** Elastic-plastic stage (from point A to point B). During this stage, concrete in the compressive zone is confined by the steel tube because the Poisson ratio of concrete becomes larger than that of steel. The confinement enhances as the longitudinal deformation increases. The stiffness decreases with the increase of the zone of steel yielding as a result the moment ($M$) versus rotation angle ($\theta$) curves tend to go upwards. The shape of the curve mainly depends on the value of axial load level. When this level is small, the curve goes up steadily to point B, while when the level is relative big, the curve starts to go down after a short increase to point B. In other words, the smaller the axial level, the later the curve starts to fall down.

**Stage 3:** Unloading stage (from point B to point C). During this stage, moment ($M$) versus rotation angle ($\theta$) response shows linear behavior.

**Stage 4:** Elastic curve of reverse loading (from point C to point D). During this stage, the moment ($M$) versus rotation angle ($\theta$) response shows nonlinear behavior. Generally, the steel in the outer fiber goes into yielding from point D.
Stage 5: Elastic-plastic stage of reverse loading (from point D to point E). During this stage, the moment (M) versus rotation angle (θ) response nonlinear behavior. The stiffness of the columns decreases with the increase of the steel yielding zone, as well as the tensile zone in the cross sections.

Stage 6: Reloading stage (from point E to point F). During this stage, the moment (M) versus rotation angle (θ) response shows similar behavior as the stage from point B to point E.

4. PARAMETRIC STUDIES

4.1 Experimental investigation

Table 1. shows 48 circular CFT columns with different diameter to thickness ratios (D/t), steel tube strength (f_y) and concrete strength (f_c) under a cyclic load protocol with variable intensity. Eurocode 4 [EC4] provides a range from 20 to 50 MPa for the concrete strength, 235 to 460 MPa for the steel strength and for the slenderness ratio (D/t) a value ≤ 90e^a, where e = √235/f_y.

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Table 1: Specimen dimensions and material properties for the parametric study.
The lateral loading history is based on Applied Technology Council (ATC) 24 guidelines [11] for cyclic testing of structural steel components, as modified in the test procedures which were conducted by Inai et al. [4]. Figure 4 shows a part of the complete lateral loading history, which consists from the following several displacement levels: 0.25\(u_y\), 0.50\(u_y\), and 0.70\(u_y\) for elastic cycles and 1.0, 1.5, 2.0, 3.0, 5.0, 7.0, 8.0, 11.0 and 14.0\(u_y\) for inelastic cycles with three cycles imposed at each displacement level of 1.0, 1.5, 2.0\(u_y\) and two cycles at other levels.

**Figure 4.** Lateral loading history.

The loading history consists of lateral elastic and inelastic displacement cycles adding an axial constant load of 0.20\(P_0\), where \(P_0 = A_a f_a + A_c f_c\) is the axial load capacity of the columns. According to EC4 [12], the effective secant stiffness \(K_{eff}\) of specimen is given by:

\[
K_{eff} = 3 \frac{EI_{eff}}{L^3}
\]

where \(L\) is the total height of specimen and \(EI_{eff}\) is the effective flexural stiffness of a cross section of a composite column with the following form [12]:

\[
(EI_{eff}) = K_0(E_a I_a + K_e E_{cm} I_c)
\]

where \(K_e\) is a correction factor which should be taken as 0.6, \(K_0\) is the calibration factor which should be taken as 1.0 and \(E_{cm}\) is the secant modulus of elasticity of concrete in GPa which can be evaluated by the following formula:

\[
E_c = 22 \left(\frac{f_c + 8}{10}\right)
\]

Finally, \(I_a\) and \(I_c\) are the second moments of area of the structural steel section and the un-cracked concrete, respectively where can defined as follows [12]:

\[
I_a = \frac{\pi d_a^4}{64}, \quad I_c = D^4 - 2d_c^4, \quad I_a = \frac{\pi}{64}(D^4 - d_c^4)
\]

where \(D\) is the external diameter of steel tube, \(d_c\) is the diameter of the concrete core and \(t\) the thickness of steel tube. Furthermore, indices “a” and “c” have to do with steel and concrete regions. The horizontal yield force \((H_y)\) of the specimen can be determined by the corresponding yield moment \((M_y)\), as [12]:

\[
H_y = \frac{M_y}{L}
\]

### 4.2 Analytical interaction relation between axial force and bending moment

In this section, a polynomial expression is developed in order to represent the two-dimensional axial force-bending moment \((N-M)\) cross-section strength for circular CFT columns having a wide range of material strengths and cross-section dimensions. The \(N-M\) interaction curve for circular CFT columns is represented by the polynomial equation [13]:
The horizontal yield displacement ($\Delta_y$) for Eq. (13) and its corresponding axial force $N_{max}$ and its maximum pure bending moment $M_0$ as shown in Figure 5. The above constants are given by:

$$k_1 = M_0$$

$$k_2 = \frac{(2N_{max}^4 - 4N_{max}^2N_{M_{max}}^2)(M_{max} - M_0) - 2M_0N_{M_{max}}^2}{N_{max}N_{M_{max}}(2N_{M_{max}}^2 - 3N_{max}N_{M_{max}}^2 + N_{max}^3)}$$

$$k_3 = \frac{(4N_{max}N_{M_{max}}^3 - N_{max}^4)(M_{max} - M_0) + 3M_0N_{M_{max}}^4}{N_{max}N_{M_{max}}^2(2N_{M_{max}}^2 - 3N_{max}N_{M_{max}}^2 + N_{max}^3)}$$

$$k_4 = \frac{(N_{M_{max}}^2 - 2N_{max}N_{M_{max}})(M_{max} - M_0) - M_0N_{M_{max}}^4}{N_{max}N_{M_{max}}^2(2N_{M_{max}}^2 - 3N_{max}N_{M_{max}}^2 + N_{max}^3)}$$

In this paper, the four characteristic quantities of interaction curve result from empirical equations as

$$N_{max} = A_c f_c (a_1 + a_2 \frac{t}{D} + a_3 \frac{f_c}{f_y} + a_4 \theta)$$

$$M_{max} = W_c f_c (b_1 + b_2 \frac{t}{D} + b_3 \frac{f_c}{f_y} + b_4 \theta)$$

$$N_{M_{max}} = A_c f_c (d_1 + d_2 \frac{t}{D} + d_3 \frac{f_c}{f_y} + d_4 \theta)$$

$$M_0 = W_c f_c (c_1 + c_2 \frac{t}{D} + c_3 \frac{f_c}{f_y} + c_4 \theta)$$

$$\theta = \frac{A_s f_c}{A_c f_c}, \quad W_c = \frac{\pi d_i^3}{32}$$

where $W_c$ is the elastic section modulus of concrete core in $m^3$, the $a_i, b_i, c_i$ and $d_i$ factors ($i=1-4$) of Eqs (16)-(19) are shown in Table 2. The resulted axial forces and bending moments are expressed in KN and KNm, respectively. Furthermore, the diameter $D$ and the thickness $t$ should be expressed in $mm$, the steel yield stress ($f_c$) and the concrete compressive strength ($f_y$) in MPa, section areas of steel ($A_s$) and concrete ($A_c$) in $m^2$.

The horizontal yield displacement ($\Delta_y$) can be defined using the Eqs (13) and (9), as $\Delta_y = H_y/K_{eff}$. The effectiveness of Eq. (13) for $H_y$ and $\Delta_y$ are shown in Figure 6 which compared with ATENA values.

### Table 2: Parameters a-d.

<table>
<thead>
<tr>
<th>Subindex</th>
<th>$N_{max}(a_1)$</th>
<th>$M_{max}(b_1)$</th>
<th>$M_0(c_1)$</th>
<th>$N_{M_{max}}(d_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>817.2</td>
<td>595.2</td>
<td>22.50</td>
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</tr>
<tr>
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<td>5528</td>
<td>5924</td>
<td>-6358</td>
<td>1961</td>
</tr>
<tr>
<td>3</td>
<td>128.3</td>
<td>275.5</td>
<td>6.827</td>
<td>108.4</td>
</tr>
<tr>
<td>4</td>
<td>1282</td>
<td>2103</td>
<td>-2257</td>
<td>213.9</td>
</tr>
</tbody>
</table>

**Figure 5.** Typical axial force-bending moment interaction curve.

$$k_1 + k_2N + k_3N^2 + k_4N^4 - M_y = 0$$ (14)
5. PARAMETERS DETERMINATION OF RAMBERG-OSGOOD MODEL

5.1 Modified Ramberg-Osgood hysteretic model

The Ramberg-Osgood model is used to describe load-displacement hysteresis curves $H(\Delta)$ displaying an elastic branch up to the yield displacement $\Delta_y$ and the corresponding yield force $H_y$, followed by a transition curve which leads to a plastic branch. The transition between the elastic and plastic branch, is controlled by the Ramberg-Osgood factor $r_2$ whose influence is presented in Figure 7. For monotonic loading, the modified Ramberg-Osgood hysteresis model is expressed by the following form [3]:

$$\Delta = \frac{H}{K_0} \left[ 1 + \left( \frac{H}{nH_y} \right)^{r_2-1} \right]$$

(21)

For cyclic loading, the equation of Ramberg-Osgood hysteresis model will be:

$$(\Delta \pm \Delta_i) = \left( \frac{H \pm H_i}{K_0} \right) \left[ 1 + \left( \frac{H \pm H_i}{nH_y} \right)^{r_2-1} \right]$$

(22)

where $\Delta$ and $\Delta_i$ are the displacements, $H$ and $H_i$ are the lateral loadings, $H_y$ is the effective first yield, $K_0$ the initial elastic stiffness and $n$ is the coefficient where $n$ is ranged from 1 to 2. In Figure 7, monotonic loading represents the initial loading, which is the path 1-2 with $n$ equal to 1. On the other hand, Eq. (22) is valid only for cyclic loading, which is the path 2-3-4 with $n$ equal to 2. Figure 8 shows various values of $r_2$ which includes as limiting cases the elastic ($r_2 = 1$) and elastoplastic ($r_2 = \infty$) relations [14].

5.2 Calibration of Ramberg-Osgood hysteretic model

The Ramberg-Osgood hysteresis model which is also available in Ruaumoko program [6], can be used to simulate the hysteretic behavior of circular concrete-filled steel tube columns. The parameters of this method are: the effective stiffness ($K_{eff}$), post-yield stiffness ratio $r_2$ as well as the positive and negative yield force $H_1$ and $H_2$, respectively. The post-yield stiffness ratio, the positive and negative yield force can be defined as:
\[ r_2 = 4 + \frac{f_y - 235}{225}, \quad H_1 = H_y a, \quad H_2 = -H_1 \] (23)

Factor \((a)\) can be determined by the following expressions:

\[ a = \left( k_1 + k_2 f_y \right) + \left( k_3 + k_4 f_y \right) \left( \frac{D}{t} \right)^3 + \left( k_5 + k_6 f_y \right) f_c, \quad \text{D/t} \leq \text{max (D/t)} \] (24)

where \(k_1 = 1.207, k_2 = 3.231 \times 0.04, k_3 = 1.800 \times 0.06, k_4 = -6.328 \times 0.09, k_5 = -6.894 \times 0.03\) and \(k_6 = 8.946 \times 0.06\). The steel yield stress \((f_y)\) and the concrete compressive strength \((f_c)\) are expressed in MPa. Figure 9, illustrates the differences between calibration of Ramberg-Osgood model and finite element analysis model in three cases of specimen with the same \(f_c\) (20 MPa) and \(f_y\) (235 MPa) but different \(D/t\).

**Figure 9.** Calibration of Ramberg-Osgood model in parametric study.

As can be seen from the above three cases, impairment and stiffness phenomena do not appeared for IDR=5% as displayed in the case of square columns [3]. Also, the results from this method are in very good agreement with those obtained from ATENA program, thus confirming the validity of the proposed method. Moreover, the proposed method demonstrates good numerical performance for the behavior of circular CFT columns under the combination of axial force with bending moment. The accuracy of the proposed model is confirmed by comparing its results with those of experiments of Inai et al. [4] for circular CFT columns under a cyclic load protocol with variable intensity and constant axial load as shown in Figs 10 and 11.

**Figure 10.** Calibrated of R-O model against experimental data (Inai et al.[3] – SC4-A4C and SC8-A4C).
6. CONCLUSIONS

This study develops a proposed method which predicts the cyclic behavior and strength of circular concrete-filled tube columns under axial and bending moment. Also, it outlines the construction of a simple and accurate hysteretic model for simulating the cyclic behavior of circular CFT columns. It worth noticing that, generally, circular CFT columns do not show deterioration phenomena such in case of square columns. Hence, one can successfully use this simple calibrated Ramberg-Osgood model for CFT columns either for individual element or as members of composite MRFs frames in order to determine their seismic response easily and reliably.

7. REFERENCES