8th GRACM International Congress on Computational Mechanics,
Volos, 12 July - 15 July 2015

EFFECT OF SUPPORT CONDITIONS ON ESTIMATING HANGER TENSION IN ARCH BRIDGES USING MODAL FREQUENCY MEASUREMENTS

K. Giakoumi[1], C. Papadimitriou[1], C. Argyris[1], L. Spyrou[2] and P. Panetsos[3]

[1]Department of Mechanical Engineering
University of Thessaly
Volos, GR-38334, Greece
e-mail: {kogiakou,costasp,koargiri}@uth.gr; web page: http://www.mie.uth.gr/labs/sdl

[2]Centre for Research and Technology Hellas (CERTH)
Institute for Research and Technology - Thessaly
Volos 38333, Greece
e-mail: lspyrou@ireteth.certh.gr; web page: http://www.ireteth.certh.gr

[3]Capital Maintenance Department
Egnatia Odos S.A.
Thermi 57001, Greece
e-mail: ppane@egnatia.gr; web page: http://www.egnatia.gr

Keywords: Bayesian Inference, Hanger Tension Estimation, Modal Frequencies, Geometric Nonlinearities.

Abstract: *The axial loads of bridge hangers are identified using experimentally estimated modal frequencies. Two different categories of model classes are considered to simulate the vibrations of the hangers: an analytical model based on the Euler-Bernoulli theory, and a high fidelity finite element model of the hanger. The effect of the boundary conditions of the hangers on the estimate of the axial loads and their uncertainties is investigated. A Bayesian parameter estimation and model selection method is used to estimate the axial load and its uncertainty. It is demonstrated that the boundary conditions have a significant impact on the frequencies of hangers used in arch bridges. It is also shown that a fixed-end high fidelity finite element model of the hanger underestimates the axial loads by more than 20%. The simplified beam model with flexible end conditions gives fairly accurate results, close to the ones obtained from the high fidelity finite element model with flexible support conditions.*

1 INTRODUCTION

Hangers are used as deck support elements in arch bridges. Methods to monitor the axial loads in hangers are important for identifying the structural integrity of arch bridges. When the axial loads are large enough so that they affect the modal frequencies due to the stiffness increase, their estimation is based on comparing model predictions with the experimentally estimates of modal frequencies. The axial load is then estimating as the one that gives model predictions that matches the experimental modal frequency values. However, the predictions of the model frequencies may be affected by the boundary conditions at the end supports of the hangers. This study examines the effect of the model and the boundary conditions on the hanger modal frequencies and the axial load estimation.

Several methods have been proposed for the estimation of the cable tension. Zui et al. [1] proposed practical formulas for the estimation of the tension from the identified natural frequencies. Ren et al. [2] presented a new version of the practical formulas, after having explained the relative influence of the sag-extensibility and the bending stiffness. Ni et al. [3] proposed a method for the analysis of a suspended bridge and a stay-cable bridge. Ceballos and Prato [4] introduced two rotational springs at the cable ends to represent all the possible boundary conditions and the stiffness of the springs is extracted from the first mode that cannot be estimated by the identification procedure. Successively, the axial force of the cable can be calculated. Bellino et al. [5] proposed a method to estimate the cable tension by means of vibration response and moving mass technique. The same authors developed the method [6] by adding springs in order to simulate the different boundary conditions and they introduced the equivalent length.

The objective of this work is to estimate the axial loads in hangers using modal frequencies identified from modal tests. Two different categories of model classes are introduced to represent the dynamics of the hangers. The first category is based on a conventional Bernoulli-Euler beam models. Two sets of boundary conditions are considered, one with fixed ends and the other with flexible ends quantified by rotational springs attached at the ends. The second category is based on high-fidelity finite element models of the hanger developed in Abaqus. For high values of the axial load, large deflections are obtained which introduce geometrical nonlinearities. These nonlinearities increase the stiffness of the structure at the equilibrium state and thus affect the modal frequencies. In fact the modal frequencies are obtained by solving the geometrically nonlinear problem for a hanger subjected to large axial loads and then using the tangent stiffness matrix at the final equilibrium position to estimate the modal frequencies. Three different types of boundary conditions are implemented and the effect axial load estimation is evaluated. The first type assume fixed ends, the second type models the end surfaces as flexible with respect to the rotation about axes perpendicular to the hanger axis, and the third type assumes the ends to be flexible in translational motion about the perpendicular axis of the hanger. The flexibility of the end supports is appropriately modeled by attaching springs.

This work investigates the effect of the model assumptions and the boundary conditions on the estimation of the hanger axial forces. Bayesian inference [7], [8], [9] for parameter estimation and model selection is used for estimating the axial force and boundary conditions of the hanger based on the different model classes introduced, as well as the experimentally identified modal frequencies. Bayesian model selection method [10] is used to select the best model class for representing the dynamics of the hangers. In contrast to existing methods, the present work uses Bayesian inference and more than one modal frequencies to estimate the axial force and the boundary conditions. Bayesian inference [7] is used for the first time to estimate also the uncertainty in the estimates of the axial force and the boundary conditions as well as to select the best model out of a series of increasingly complex models, including analytical ones as well as high fidelity finite element models with various boundary conditions introduced to represent the support conditions of the hanger. The Transitional Markov Chain Monte Carlo (TMCMC) algorithm [7] is used to provide estimates of the axial forces and the boundary conditions, along with the associated uncertainty in their values. Results are presented for five model classes with fixed and flexible boundary conditions. The results are compared and useful conclusions are obtained related to the estimate of the hanger axial force along with its uncertainty, the effect of edge supports and boundary conditions on the prediction of the axial load, as well as the adequacy of each one of the five model class to represent the dynamics of the hanger.

2 Description of Hanger

The hanger under investigation is one of the 20 variable-length hangers used to support the deck of the arch bridge shown schematically in Figure 1. The hanger geometry, along with the geometry of the connections of the hangers to the deck and the arch, is shown in Figure 2a. They are made out of steel and they are connected to the deck and the arch element with edge plates as shown in Figure 2b. The edge plates are welded to the hangers and the deck or the arch. The connections of the hanger with the deck or the arch, with end plates are approximately 1 m long and may affect significantly the boundary conditions of hangers. The plate that connects the hanger with the deck has its orientation along the longitudinal direction of the bridge deck, while the plate that connects the hanger to the arch has its orientation along the transverse direction of the bridge deck.

The hangers are made of steel with modulus of elasticity $E = 200Gpa$, mass density 7800 kg/m^3 and Poisson ratio 0.3. The total length of the hanger, including the connections is approximately 12 m, while the length of the circular section of the hanger is 9.817 m. The diameter of the circular section of the hanger is 0.13 m. Based on the geometry of the hanger's cross-section, the bending stiffness of the present hanger affects the modal frequencies. In addition the solid plate elements installed in there hangers to connect the circular section to the bridge arch and deck also contribute significantly to the bending stiffness. Based also on the design plans, the connection details of the two edge plates of the hanger are identical. The only difference is the orientation of each edge plate and the geometrical characteristics of the arch and deck plates on which the connection plates are attached to. Assuming that the end conditions of the edge plates are fixed, the prediction of the modal frequencies of vibration of the hanger predicted from either the analytical or the numerical models along the longitudinal and transverse direction bridge are expected to be identical.

Table 1 shows the experimentally identified modal frequencies of the third hanger in the transversal and longitudinal direction. These modal frequencies were estimated by analyzing the force and acceleration time histories obtained from impulse hammer tests performed on the hanger. It is observed that the modal frequencies differ in the two longitudinal and transverse directions. The percentage differences, shown in Table 1, range from 3.07% to 6.41% and cannot be justified by material or geometric variability of the hanger. Due to the symmetry of the hanger and the edge plates, this is a strong indication that the boundary conditions at the end of the hangers are responsible for such differences. Thus the hanger end conditions cannot be assumed to be fixed. This study investigates the effects of the boundary conditions on the estimation of the axial force and provide evidence, based on Bayesian inference, that fixed boundary conditions assumed for the ends may result in misleading estimated of the axial hanger loads.

3 Modal Frequency Predictions of Hangers based on Beam Theory

The Euler-Bernouli beam theory is used to predict the modal frequencies of the hanger subjected to an axial load. The equation of motion of the Euler-Bernoulli beam subjected to axial tension T [11] is given by:

$$EI\frac{\partial^4 u}{\partial^4 z} - T\frac{\partial^2 u}{\partial^2 z} + \rho A\left(\frac{\partial^2 u}{\partial t^2}\right) = 0 \tag{1}$$

where $u(z,t)$ is the displacement/deflection of the beam in the transverse direction in the (x,z) plane, ρ is the density, E is the modulus of elasticity, I is the moment of inertia of the circular cross section about the x and y axes, A is the area of the cross-section of the beam. All geometrical and material properties are assumed constant along the length of the beam. Two model classes are introduced that differ on the boundary conditions considered. In the first model the ends of the beam are considered fixed, whereas in the second model the ends are considered flexible. The flexibility (support stiffness) in rotation of the ends is simulated using rotational springs at the ends.

For fixed-end supports, the boundary conditions are $u(0,t) = 0$, $u(L,t) = 0$, $u'(0,t) = 0$, $u'(L,t) = 0$. Following the usual eigenvalue analysis, the modal frequencies are obtained by solving the characteristic equation. This is an analytical transcendental equation which is numerically solved to obtain the modal frequencies.

For flexible supports, modeled by rotational springs at the two ends, the boundary conditions for beam deflections in the (x,z) plane are $u(0,t) = 0$, $EIu''(0,t) - k_1 u'(0,t) = 0$, $u(L,t) = 0$ and $EIu''(L,t) + k_3 u'(L,t) = 0$, where k_1 and k_3 are the rotational springs applied at the ends to resist rotation around y direction due to bending. Similar support conditions hold for beam displacements in the (y,z) plane with k_2 and k_4 introduced as rotational springs applied at the ends to resist rotations with respect to the x direction.

Two model classes are introduced based on the Euler-Bernoulli beam theory. Model class *Beam-Fixed* is the beam model with fixed ends, while model class *Beam-Flex* is a beam model with flexible ends simulated by rotational springs, two at each side of the beam. For the three-dimensional beam, the vibrations of the beam in the (x,z) and (y,z) planes are considered uncoupled for both model classes. Thus, for a given value of the axial load the modal frequencies are computed by solving 2 two-dimensional beam problems, considering the boundary conditions for each plane motion, one in the (x,z) with spring constants k_1, k_3 and the other in the (y,z) plane with spring constants k_2, k_3. Thus we are able to obtain different frequencies along the transverse and longitudinal direction of the beam by applying the different spring constants.

4 Modal Frequency Predictions of Hangers based on Finite Element Models

High fidelity finite element models are also used to predict the modal frequencies of the hanger under various boundary conditions. Abaqus is used to construct the finite element models and perform the analyses required. The finite element model is denoted by FE and uses Timoshenko beam elements to create the mesh in the $10m$ beam section with circular cross-section, while the ten-node tetrahedral element (C3D10) element are used to mesh the two $1m$ end connections of the circular beam with the rest of the bridge structure.

4.1 Types of Boundary Conditions and Model Classes

Three different types of boundary conditions are considered. The first type assumes fixed ends at the boundaries and is implemented by constraining the motion of the DOFs at the edge surfaces. The second type permits only the rotation of the bottom and top edge surfaces of the hanger plate connectors, about the two axes x and y along the longitudinal and transverse directions of the bridge deck, respectively. These boundary conditions are implemented by constraining the motion of the midpoint of the edge surface to zero along all three x, y and z directions, constraining the movement of the side nodes of the edge surface along the x and y directions to zero, and adding springs along the z direction of the side nodes of the edge surface restraining their motion along the z direction of the hanger according to the spring constants. Such springs provide resistance to rotations of the edge surfaces along the x and y axes. Two independent sets of springs are added to simulate the rotational resistance along the x and y axes. Each set is uniformly distributed along the opposite sides of the edge surface. The third type of boundary conditions permits the translation of the edge surfaces along the x and y directions in addition to the rotation as described in the second type. The boundary conditions are implemented by fixing the constraining the motion of the center node of the edge surface along the z axis, adding the springs as described in the second type, and adding two sets of uniformly distributed springs at the opposite sides of the edge surface along the the x and y axes, respectively as well as at the mid node of the edge surface.

Model classes $FE-i$, $i=1,2,3$, are introduced as high fidelity finite element models of the hanger. The i stands for the type of the boundary condition assumed. Specifically, the following model classes are employed which depend on the boundary conditions considered.
1. Model class $FE-1$: Uses type 1 (fixed-ends) BC.
2. Model class $FE-2$: Uses type 2 BC, allowing only rotation about the x and y axes at both bottom and top hanger ends.
3. Model class $FE-3$: Uses type 3 BC, allowing translation along and rotation about the x and y axes at both bottom and top hanger ends.

4.2 Estimation of Modal Frequencies

The modal frequencies of the hanger are affected by the high tension which increases the tangent stiffness. In order to predict the tangent stiffness increase due to tension and subsequently the effect on the modal frequencies of the hanger with axial load, a geometrically nonlinear analysis of the beam has to be performed. The evaluation of the modal frequencies in Abaqus that take into account the stiffness increase due to large axial loads consists of a certain sequence of actions as follows.
Action 1: This involves is a static deformation for estimating the tangent stiffness matrix under the application of the axial load. The hanger nodes at one edge (e.g. the top edge) are considered fixed along the longitudinal direction and free along the transverse directions of the hanger and the axial load is applied to the bottom edge. The axial load T is simulated as a pressure $p = T/(bh)$ uniformly distributed through the bottom face of the connection. The geometrically nonlinear static analysis is performed and the tangent stiffness matrix and the mass matrix are then extracted from the abaqus model.
Action 2: The correct boundary conditions are applied at both edges. After the hanger has been elongated in Action 1, the boundary conditions at the top and bottom edges are activated and the hanger is kept in its deformed state. Specifically, for fixed-end hanger conditions, all displacement DOFs at the bottom and top edges of the hanger are restrained. For flexible supports, springs are added at the top and bottom edge DOFs so that the translational and rotational stiffness conditions are simulated. The resulting mass matrix as well as the tangent stiffness matrix which consist of the tangent stiffness matrix and the spring constants is extracted from Abaqus.
Action 3: Using the mass matrix and the tangent stiffness matrix, modified to include the boundary states at the top and bottom edges of the hanger, the eigenvalue analysis is performed to obtain the modal frequencies of the hanger.

5 Bayesian Inference for Model Selection and Parameter Estimation

The Bayesian framework for parameter estimation and model selection [8], [9] is used to estimate the hanger axial load and the boundary conditions based on the linear and nonlinear model classes introduced in the previous section. The inference is based on the lowest m experimentally identified modal frequencies $D = \{\omega_r, r = 1, ..., m\}$ of the hanger. Consider a parameterized class of models used to model the dynamic behavior of the hanger and let $\underline{\theta} \in R^{N_\theta}$ be the set of free structural model parameters to be identified. Let also $\omega_r(\underline{\theta})$ be the predictions of the modal frequencies obtained for a particular value of the parameter set by solving the corresponding eigenvalue problem for the model class considered.

Using Bayes theorem, the updated (posterior) probability distribution $p(\theta|D, M)$ of the model parameters θ based on the data D, is obtained as follows:

$$p(\underline{\theta}|D, M) = \frac{p(D|\underline{\theta}, M)p(\underline{\theta}|M)}{p(D|M)} \tag{2}$$

where $p(D|\underline{\theta}, M)$ is the likelihood function, $p(\underline{\theta}|M)$ is the initial (prior) probability distribution of a model, and $p(D|M)$ is the evidence of the model class given by

$$p(D|M) = \int_{\Theta} p(D|\underline{\theta}, M)p(\underline{\theta}|M)d\underline{\theta} \tag{3}$$

The likelihood $p(D|\underline{\theta}, M)$ is derived by using a probability model for the prediction error e_r for the r-th modal frequency defined as the fractional difference between the measured modal frequency and the corresponding modal frequency predicted from a model using a particular value of the parameter set $\underline{\theta}$. Specifically, e_r satisfies the prediction error equation

$$\widehat{\omega}_r = \omega_r(\underline{\theta}) + \widehat{\omega}_r e_r \tag{4}$$

for all modes $r = 1, ..., m$. Modeling the predictions errors as zero-mean Gaussian variables, $e_r \sim N(0, \sigma^2)$, with standard deviation σ, assuming that the prediction errors are independent, and including the prediction error parameters σ into the uncertain parameter set $\underline{\theta}$, the likelihood $p(D|\underline{\theta})$ takes the form

$$p(D|\underline{\theta}) = \frac{1}{(\sqrt{2\pi})^m \sigma^m} exp\left[-\frac{m}{2\sigma^2} J(\underline{\theta})\right] \tag{5}$$

where $J(\underline{\theta})$ given by

$$J(\underline{\theta}) = \frac{1}{m} \sum_{r=1}^{m} \frac{[\omega_r(\underline{\theta}) - \widehat{\omega}_r]^2}{[\widehat{\omega}_r]^2} \tag{6}$$

represents the measure of fit between the measured modal frequencies and the modal frequencies predicted by the model.

The Bayesian framework can be used to select the best model class among a family of alternative model classes. Specifically, let $M_1, \ldots, M_\mu$, be competitive model classes used to represent the dynamics of the hanger. Using the Bayes theorem, the posterior probability $P(M_i|D)$ of the model class M_i given the data D is obtained from

$$P(M_i|D) = \frac{p(D|M_i)P(M_i)}{P(D)} \tag{7}$$

where $p(D|M_i)$ is the evidence of M_i, $P(M_i)$ is the prior probability of M_i and $P(D)$ is a normalizing constant that guaranties that the sum of the probabilities over all model classes considered in the selection equals to one. Assuming that the model classes are equally probable prior to the use of the data, then the most probable model class based on the data corresponds to the model class with the highest evidence [9].

Bayesian computational tools are used to estimate the uncertainty in the model parameters and select the best model class. Specifically, the Π4U software [12] is used for parameter estimation, model selection and uncertainty quantification. Herein we use the TMCMC algorithm as proposed in Ching and Chen [7] and its parallelized and extended version in [9] and [12] in order to sample the posterior PDF and propagate uncertainties to compute output quantities of interest such as the axial hanger load and the boundary stiffness. One more merit of using the TMCMC algorithm for Bayesian purposes is the calculation of the evidence is a by-product of the algorithm. The most probable values of the parameters in $\underline{\theta}$ are obtained by minimizing the $-logp(\underline{\theta}|D, M)$ using the CMA-ES algorithm in $\Pi 4U$ software.

6 Numerical Results

The axial load is estimated using the five model classes introduced in the previous sections and the lowest twelve experimentally identified modal frequencies, six along the transverse and six along the longitudinal direction of the bridge deck. The objective is to estimate the axial load and its uncertainty, to explore the effect of the hanger end conditions on the axial load and to select the best model classes that are adequate representations of the hanger behavior.

The nominal value of the axial load is considered to be $P_0 = 921,88KN$ corresponding to the most probable value of the fixed-end beam model $Fixed - Beam$ based on the lowest two experimental frequencies, one in the transverse and the other in the longitudinal direction. The nominal value of the hanger length is $L_0 = 12m$, corresponding to the total length that includes the length along the hanger direction of the plate connectors at the two ends. The parameters in the set $\underline{\theta}$, introduced for the axial load P and the length L of the beam, multiply the nominal values of the respective parameters so that the axial load is $P = \theta_P P_0$ and the length is $L = \theta_L L_0$. The variation of the boundary stiffness values is unknown. In the Bayesian estimation process the range will be left to vary from 10^5 to 10^{15}. The nominal values of the spring stiffness are taken to be $k_{nom} = 10^{10}$ for both translational and rotational stiffness values. For a boundary spring stiffness k of the hanger, the corresponding parameter θ is given as $k = k_{nom}^{\theta} = 10^{10\theta}$ so that the θ values range from 0.5 to 1.5.

The prior distributions for all parameters are selected to be uniform with values ranging from 0.5 to 1.5 ($\theta_i \in [0.5, 1.5]$). The range of variation of the prediction error parameter σ is $\sigma \in [0.001, 0.1]$. The Π4U software [12] is used for parameter estimation, model selection and uncertainty propagation. Results are obtained using the TMCMC algorithm which samples the posterior PDF of each model class, computes the uncertainties in the model parameters, and estimates the evidence of the model class. The values of the TMCMC parameters [7] were selected to be $\beta^2 = 0.2$ and $tolCov = 1.0$. The most probable values of the model parameters are obtained using the CMA-ES algorithm in 4U software. The search domain is the one defined by the support of the uniform priors assumed for the model parameters.

The parameter estimation and the model class selection is performed for the following model classes reported in Table 2. For each model class, Table 2 summarize the information on the number of model parameters involved and the number of samples per TMCMC stage.

Figure 3 presents the propagation of the uncertainty in the model parameters to the output modal frequencies of the model. The results in the Figure correspond to predicted modal frequencies normalized with respect to the experimental frequencies. Thus the distance of these values from one is a measure of how close the values of the model predicted modal frequencies are to the experimentally identified modal frequencies. We can clearly see the impact of the flexible end supports in predicting the modal frequencies compared to the fixed-end model class $FE - 1$. We also notice that the predicted modal frequencies of these model class $FE - 2$ are very close to those of model class $FE - 3$. Thus the translational springs of model class $FE - 3$ in x and y direction do not have significant impact in the performance of the model.

The evidence values $P(D|M)$ for each model class, calculated by the TMCMC algorithm, are presented in Table 3. According to equation (7) the most probable (suitable) model class for the given data D, with the largest evidence, is the model class $FE - 2$ which is the high-fidelity finite element model with the four rotational springs. It is important to note that even though model class $FE - 3$ gives essentially the same fit with model class $FE - 2$, it has a smaller evidence associated with it due to over parameterization. The evidence $P(D|M)$ defined in equation (3) takes into account the number of parameters of the model, and penalizes the model with the more parameters that yields similar results. The $Beam - Fixed$ model performs better than the $FE - 1$ model with fixed supports due to the fact that the $Beam - Fixed$ has more freedom to fit the data due to the effective length parameter that is free to be determined by the modal frequency data. The $Beam - Flex$ model is the third best model that is promoted by the Bayesian methodology. Finally, the models $Beam - Fixed$, $Beam - Flex$, $FE - 2$ and $FE - 3$ provide hanger loads that differ by 2-3% and have similar uncertainties in the hanger tension values. However, the $FE - 1$ model with fixed ends underestimate the load by as much as 20%, which is an indication that this model in inappropriate to represent the behavior of the hanger due to the fact that the supports are flexible.

7 Conclusions

This paper examined methods of estimating the tension in bridge hangers via measured modal frequencies. Both analytical beam models and finite element models were developed and examined for their adequacy in representing the hanger dynamics. Moreover, a thorough analysis of boundary conditions was performed by considering both fixed and flexible hanger ends. The most accurate model for tension estimation in the examined hanger is the finite element model with four rotational springs. The fixed BC models are substantially less accurate than the ones that take flexibility into account. The use of spring elements is therefore deemed important in the model developing. In particular, the fixed-end high fidelity finite element model of the hanger underestimates the axial loads by more than 20%. The simplified beam model with flexible end conditions gives fairly accurate results, close to the ones obtained from the high fidelity finite element model with flexible support conditions.

It needs to be noted that even though the simplified analytical models are not as effective in simulating the hanger dynamics as their finite element counterparts, their computational time is several orders of magnitude lower. The time required for the analysis is very important, as the model has to run many times in the TMCMC algorithm, thus the selection of the most appropriate model should take into account this parameter as well. Especially the analytical beam model with flexible ends, with lower evidence that the high-fidelity finite element model with flexible ends, give very good estimates of the hanger load with accuracy within 2-3% of the one estimated from the finite element model.

The Bayesian framework for structural identification is effective in estimating the axial loads and boundary conditions in hangers as well as selecting the most appropriate model class to be used for representing the vibrational characteristics of hangers.

Acknowledgments

This research has been implemented under the ”ARISTEIA” Action of the ”Operational Programme Education and Lifelong Learning” and was co-funded by the European Social Fund (ESF) and Greek National Resources.

References

1. Zui, H., Shinke, T. and Namita, Y. (1996), ”Practical Formula for Estimation of Cable Tension by Vibration Method,” *Journal of Structural Engineering*, Vol. 122, pp. 651-656.

2. Ren, W.X., Chen, G. and Hu, W.H. (2005), ”Empirical formulas to estimate cable tension by cable fundamental frequency,” *Structural Engineering and Mechanics*, Vol. 20, pp. 363-380.

3. Ni, Y.Q., Ko, J.M. and Zheng, G. (2010), ”Dynamic analysis of large diameter sagged cables taking into account flexural rigidity,” *Journal of Sound and Vibration*, Vol. 257, pp. 301-319.

4. Marcelo, A.C. and Carlos, A.P. (2008), ”Determination of the axial force on stay cables accounting for their bending stiffness and rotational end restraints by free vibration tests,” *Journal of Sound and Vibration*, Vol. 317, pp. 127-141.

5. Bellino, A., Marchesiello, S., Fasana, A. and Garibaldi, L. (2010), ”Cable tension estimation by means of vibration response and moving mass technique,” *Mecanique et Industries*, Vol. 11, pp. 505-512.

6. Bellino, A., Marchesiello, S., Fasana, A. and Garibaldi, L. (2011), ”Tension estimation of cables with different boundary conditions by means of the added mass technique,” *UTC compiegne*.

7. Ching, J. and Chen, Y.C. (2007), ”Transitional Markov Chain Monte Carlo Method for Bayesian Model Updating, Model Class Selection, and Model Averaging,” *Journal of Engineering Mechanics*, Vol. 133, pp. 816-832.

8. Beck, J.L. and Katafygiotis, L.S. (1998), ”Updating models and their uncertainties. I: Bayesian statistical framework,” *Journal of Engineering Mechanics*, Vol. 124, pp. 455-461.

9. Angelikopoulos, P., Papadimitriou, C. and Koumoutsakos, P. (2012), ”Bayesian uncertainty quantification and propagation in molecular dynamics simulations: A high performance computing framework,” *The Journal of Chemical Physics*, Vol. 137, pp. 455-461.

10. Beck, J.L. and Yuen, K.V. (2004), ”Model Selection Using Response Measurements: Bayesian Probabilistic Approach,” *Journal of Engineering Mechanics*, Vol. 130, pp. 192-203.

11. William, T.T. (1948), *Theory of vibration with applications*, Prentice Hall, 2nd edition.

12. Hadjidoukas, P.E., Angelikopoulos, P., Papadimitriou, C. and Koumoutsakos, P. (2015), ”Π4U: A High Performance Computing Framework for Bayesian Uncertainty Quantification of Complex Models,” *Journal of Computational Physics*, Vol. 284, pp. 1-21.

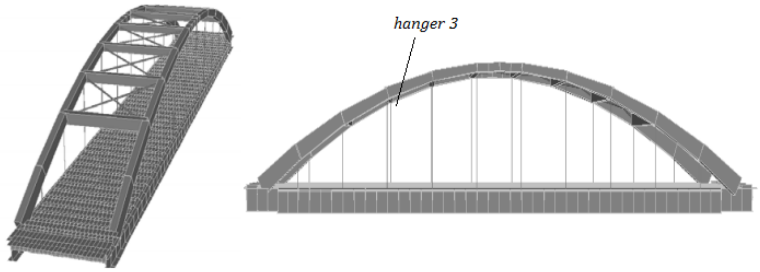
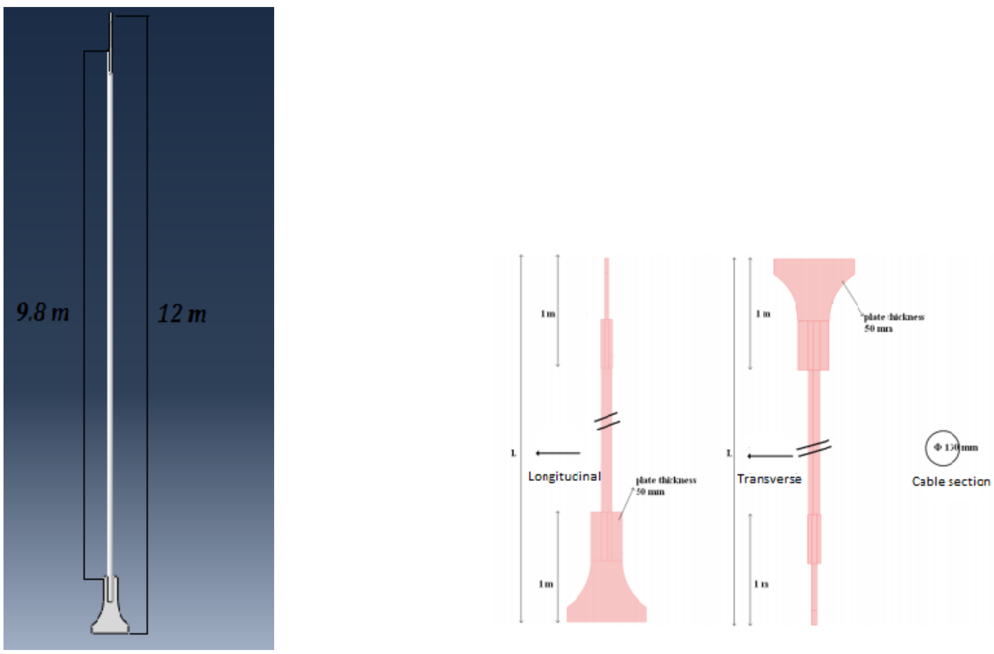


Figure 1: The geometry of the arch bridge



(a) (b)

Figure 2: (a) The geometry of hanger 3, (b) The geometry of the plates connecting the circular hanger to the arch and the deck of the bridge

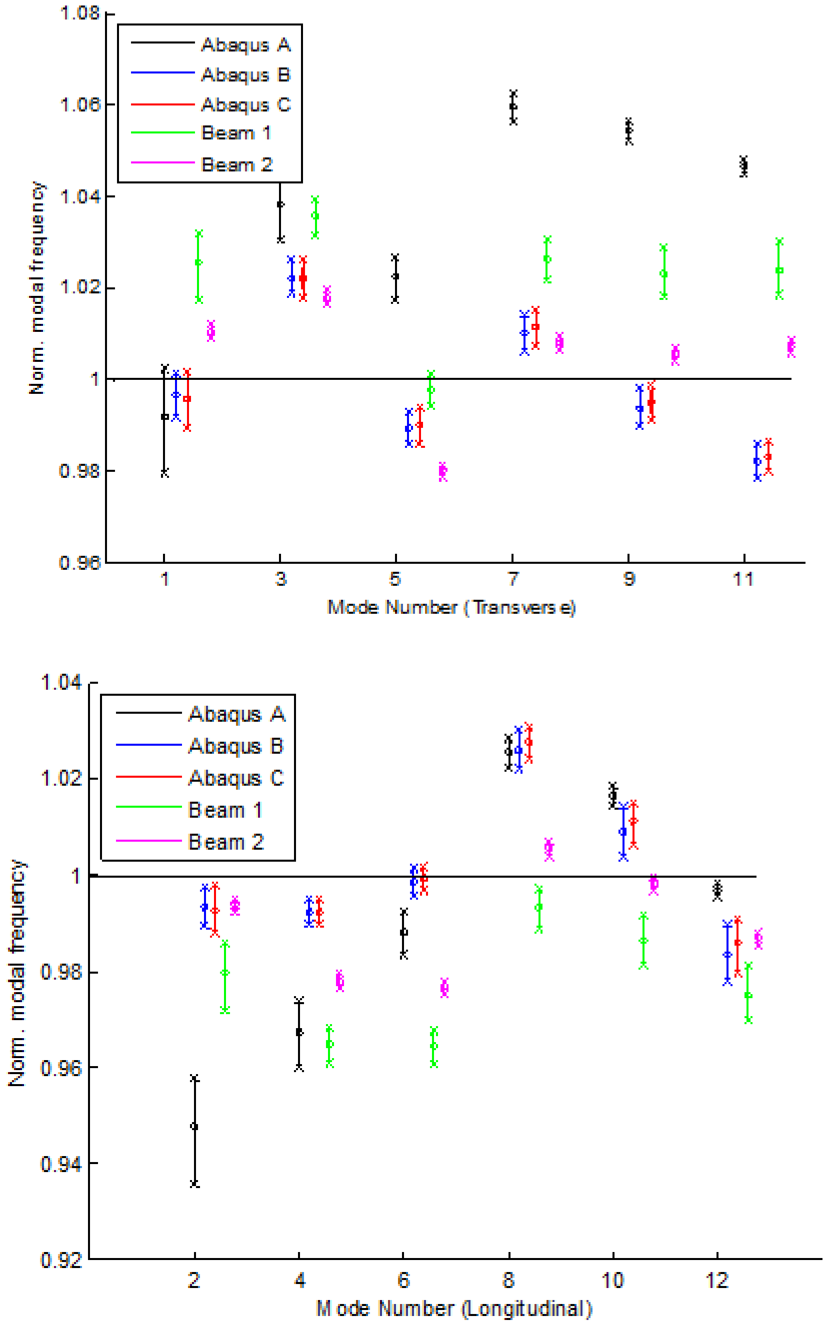


Figure 3: Uncertainty propagation to the output modal frequencies. Abaqus A$\equiv FE-1$, B$\equiv FE-2$, C$\equiv FE-3$, Beam 1$\equiv Beam-Fixed$, Beam 2$\equiv Beam-Flex$

	trans	long	% difference
Mode 1	5.82	6.09	4.43
Mode 2	13.85	14.8	6.41
Mode 3	26.17	27.0	3.07
Mode 4	40.47	41.8	3.18
Mode 5	59.3	61.5	3.57
Mode 6	81.3	83.68	2.84

Table 1: The experimental frequencies (Hz) of the hanger in the transversal and longitudinal direction

	Model Class	BC	N_θ	$N_{samples}^{TMCMC}$
1	$Beam-Fixed$	Fixed	3	500
2	$Beam-Flex$	Flex-Type 2	7	2000
3	$FE-1$	Fixed-Type 1	2	500
4	$FE-2$	Flex-Type 2	6	2000
5	$FE-3$	Flex-Type 3	10	2000

Table 2: Information of Model classes and selection of TMCMC variables

Model class	Evidence
$Beam-Fixed$	434.49
$Beam-Flex$	497.11
$FE-1$	385.04
$FE-2$	515.42
$FE-3$	512.58

Table 3: The evidence of the models