IMPLEMENTATION OF ELASTIC ENERGY RESPONSE SPECTRA SUSTAINABILITY MEASURES FOR THE PREDICTION OF DAMAGE POTENTIAL OF GROUND STRONG MOTION RECORDS

Panagiotis K. Koliopoulos[1], Emmanouil A. Kirtas[2], Georgios K. Panagopoulos[3] and Errikos K. Mouratidis[4]

Department of Civil – Surveying & Geomatics Engineering, Technological Educational Institute of Central Macedonia, Serres, Greece

[1] e-mail: kolio@teiser.gr; web page: http://civilgeo.teicm.gr/index.php?cat_id=21

[2] e-mail: kirtas@teicm.gr; web page: http://kirtas-eng.weebly.com

[3] e-mail: panagop@teicm.gr; web page http://teicm.panagop.com

[4] e-mail: erik@teicm.gr; web page: http://civilgeo.teicm.gr/index.php?cat_id=21

Keywords: Energy, Spectral sustainability, Period elongation, Damage.

Abstract. *This work explores and extends the applicability of spectral sustainability indices (presented in an earlier publication for displacement spectra), to energy response spectra. The advantage of utilizing energy spectra is that the effect of duration and amplitude-persistence of response half-cycles, is automatically incorporated into the standard 2-D energy spectra. Therefore, it is only the 'horizontal' sustainability index* λ_{E2} *that has to be introduced, taking into account the effect of the period-elongation due to expected inelastic response. Parametric time-domain inelastic dynamic analyses are performed and the computed damage indices are compared with the sustainability measures of elastic energy response spectra of selected Greek strong motion records. The results reveal the strong correlation between the proposed spectral sustainability index* λ_{E2} *and the damageability of each record. This index was further utilized to produce empirical energy-based damage predictors and compute reliable bounds of expected damage. The proposed methodology can identify the worst input scenario from a given sample of strong ground motion records, reduce the inherent errors in the production of fragility curves and facilitate the estimation of tight reliability bounds for structural systems.*

1 INTRODUCTION

One of the main challenges in performance-based seismic design is the selection of an appropriate ground motion intensity measure (IM) expressing the damage potential of the seismic action. The choice of the IM quantity is also crucial for the selection and normalization of credible earthquake ground motions needed when incremental nonlinear dynamic analyses is carried out for the construction of analytical or hybrid fragility curves [1,2].

Researchers have proposed different IM parameters based on characteristics of either the ground motion (e.g. peak ground acceleration – velocity - displacement and their effective counterparts) or the response of single degree of freedom (sdof) oscillators (e.g. response spectra ordinates at specific structural periods). It is recognized however, that due to inherent irregularities and uncertainties of earthquake occurrence mechanisms and ground properties, it is very difficult to identify a unique IM quantity which is suitable to capture the damage potential of the design seismic action [3,4].

For example, recent studies have raised serious questions regarding the efficiency and sufficiency of popular IM parameters such as the spectral acceleration at the fundamental period of a given structure. Extensive parametric nonlinear dynamic analyses indicated that there is a discrepancy between spectral ordinate values and damageability of excitation records [5,6]. This is partly due to the fact that cconventional response spectra rely exclusively on a unique instantaneous peak and do not convey information regarding duration and amplitude persistence of the secondary response cycles. In fact, it was found that ground motions with equivalent response spectra but different duration, produce substantially different collapse capacity curves (long duration reduces collapse capacity up to 40%) [6].

An additional problem stems from the elongation of the effective period when the structural response departs from the elastic range. The response spectral values at fundamental periods therefore, become increasingly inadequate to portray the structural response. To remedy this deficiency, the implementation of spectral shape parameters taking into account a range of spectral values at different periods, have been suggested [7-9]. Along these lines, in a recent publication [10] spectral sustainability measures based on 3-D displacement response spectra were introduced and proved capable not only to capture the effects of both the duration and period elongation, but also to form the basis of reliable damage predictions. Consequently, the proposed sustainability measures in conjunction with the conventional response spectral values form a promising IM quantity.

The scope of this work is to extend the aforementioned method utilising 2-D energy response spectral sustainability measures and produce reliable estimators of the damage potential of a set of design strong motion records. For sake of completeness and in order to facilitate comparisons between displacement and energy spectral indices, the basic concepts of the sustainability indices related to displacement response spectra are also presented in the following section.

2 SUSTAINABILITY INDICES OF ELASTIC RESPONSE SPECTRA

2.1 λ_1 and λ_2 indices regarding 3-D displacement response spectra

The 3-D displacement spectrum evolves from the standard 2-D response spectrum by adding a 3rd 'transversal' axis denoting the hierarchically ordered absolute maxima of the 2nd, 3rd, 4th, etc response half-cycles [11]. Half-cycles are considered in order to account for the variability of non-harmonic seismic responses. As an example, the 3-D response displacement spectrum of the longitudinal horizontal component of the Lefkas earthquake (see Table 1) is presented in Figure 1.

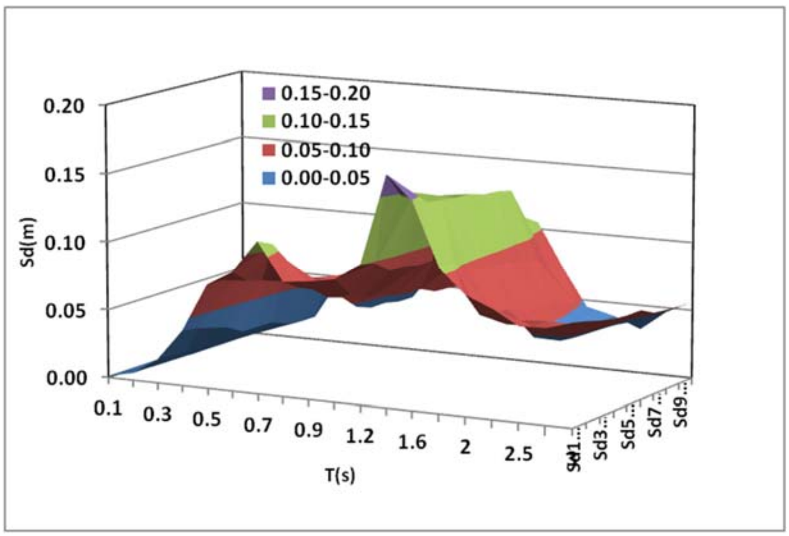


Figure 1. 3-D displacement response spectrum (LEF-L component)

The degree of persistence of the $S_1(T)$ ordinate into the secondary response peaks, can be expressed via the following non-dimensional 'transversal' sustainability index λ_1:

$$\lambda_1(T,N) = \frac{\sum_{j=1}^{N} S_j(T)}{N \cdot S_1}, \qquad \frac{1}{N} < \lambda_1 < 1 \tag{1}$$

where $S_j(T)$ is the peak of the j^{th} hieratically ordered response half-cycle, N is the number of the significant half-cycles taken into account (typically 3 – 10) and S_1 the maximum response peak (identical to the conventional 2-D spectral value).

A complementary 'horizontal' sustainability index λ_2, related to the sustainability of the elastic period's spectral ordinate $S_1(T = T_{el})$ as the response gets into the nonlinear regime and an elongation of the effective period is attained, can be defined as:

$$\lambda_2(T,\mu) = \lambda_2(T_{el}, T_\mu) = \frac{\int_{T_{el}}^{T_\mu} S_1(T_r)dT}{(T_\mu - T_{el}) \cdot S_1(T_{el})} \tag{2}$$

where T_μ is the maximum effective period (related to the target ductility μ), T_{el} is the initial elastic period, $S_1(T_r)$ is the conventional spectral value corresponding to a period $T_{el} \le T_r \le T_\mu$ and dT is the discretization step. For simple elastoplastic systems, the relation between T_μ/T_{el}, reads:

$$\frac{T_\mu}{T_{el}} = \sqrt{\mu} \tag{3}$$

A comparison between the λ_2 indices of two Greek strong motion records (see Table 1) is presented in Figure 2, for the displacement spectral ordinate at T_{el} = 0.5s and for a period elongation up to $3 \cdot T_{el}$.

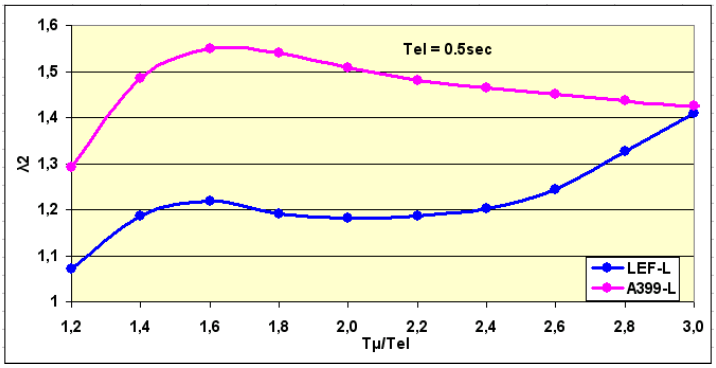


Figure 2. Comparison of displacement-spectral 'horizontal' sustainability index $\lambda_2(T,\mu)$ for a linear sdof (T=0.5s, ξ = 5%) under the action of LEF-L and A399-L records

It was shown [10] that 'transversal' sustainability index λ_1 plays a dominant role in damage accumulation for quasi-linear systems (i.e. for values of reduction factor q < 1.5), but its importance fades out as nonlinearity increases. Conversely, the importance of 'horizontal' sustainability index λ_2 rapidly rises as nonlinearity becomes stronger (q ≥ 1.5) resulting in a significant effective period elongation. Consequently, it is this index of most importance for collapse capacity studies and the rest of this work will exclusively focus on the properties and usability of λ_2.

2.2 λ_{E2} index regarding 2-D energy response spectra

The aforementioned dependency of structural damage on duration and the number of significant amplitude response half-cycles, led to the employment of the energy response spectra [12-14]. The energy balance equation of the response u(t) of a linear single degree of freedom (sdof) oscillator with mass m, damping c and stiffness k, under the action of ground acceleration $\ddot{u}_g(t)$ reads:

$$m\int_0^t \ddot{u}\dot{u}dt + c\int_0^t \dot{u}^2 dt + k\int_0^t u\dot{u}dt = m\int_0^t \ddot{u}_g \dot{u}dt \tag{4}$$

where the right-hand side term of the equation is the input energy E_{in} and the left-hand side terms correspond to the kinetic energy E_k, damping energy E_d and (elastic) strain energy E_s, respectively. Damage-related energy response spectra are normally constructed for the input energy E_{in} and the quantity $E_{in\text{-}d} = E_{in} - E_d$, termed herein as the potentially damaging energy demand.

By definition, the energy terms incorporate the effects of duration and persistence of response peaks. Hence, neither the addition of extra axis in response spectra nor the introduction of 'transversal' sustainability index is here needed. The issue of period elongation is again treated via the corresponding 'horizontal' sustainability index for 2-D energy spectra λ_{E2}, given as:

$$\lambda_{E2}(T,\mu) = \lambda_{E2}(T_{el}, T_\mu) = \frac{\int_{T_{el}}^{T_\mu} S_{E1}(T_r)dT}{(T_\mu - T_{el}) \cdot S_{E1}(T_{el})} \tag{5}$$

where S_{E1} is the energy response spectrum ordinate. The rest of the parameters are defined below equation (2).

3 CORRELATION BETWEEN SUSTAINABILITY INDICES AND COMPUTED DAMAGE

In order to investigate the correlation between the energy spectral sustainability index λ_{E2} and the damage induced by strong motion records, a series of parametric nonlinear dynamic analyses were performed for a suit of 12 Greek strong motion components, described in Table 1. This selection includes records used in previews studies regarding the Athens (1999) earthquake [13-14] and is here enriched with high energy content records of Kalamata (1986) and Lefkas (2003) earthquakes.

The nonlinear dynamic analyses comprise seven elastoplastic oscillators with elastic periods Tel = 0.2s, 0.3s, 0.4s, 0.5s, 0.6s, 0.8s, 1.0s, critical damping coefficient ξ = 5%, and twenty equally-spaced values of reduction factor q within the range $1.0 < q \leq 6.0$. Therefore, a total of 20·7 = 140 oscillators were analyzed under the 12 selected records, forming a total of 1680 cases. With the aim of demonstrating the importance of duration and spectral sustainability, the records were scaled to assume the same displacement spectral ordinate. This was done, before conducting the dynamic analyses, for each of the seven elastic periods considered.

Code	Date	Mw	Soil (EC8)	Pga(m/s^2)
KAL-L	1986	5.3	B	2.299
KAL-T	-/-	-/-	-/-	2.637
A399-L	1999	5.9	-/-	2.586
A399-T	-/-	-/-	-/-	2.972
KERT99-L	-/-	-/-	-/-	2.144
KERT99-T	-/-	-/-	-/-	1.795
SPLB1-L	-/-	-/-	-/-	3.420
SPLB1-T	-/-	-/-	-/-	3.189
SGMA1-L	-/-	-/-	-/-	1.447
SGMA1-T	-/-	-/-	-/-	2.336
LEF-L	2003	6.2	C	3.334
LEF-T	-/-	-/-	-/-	4.086

Table 1 : Characteristics of records used

Among the various global damage indices found in the literature [1], the widely accepted Park & Ang damage index is adopted herein, for the scalar quantification of damage. This index is a linear combination of the damage caused by excessive deformation and accumulated due to repeated excursions into the inelastic domain.

$$D_{PA}(T,N) = \frac{\delta_{dem} - \delta_y}{\delta_{sup} - \delta_y} + \beta \frac{E_p}{F_y(\delta_{sup} - \delta_y)}, \qquad 0 \leq D_{PA} \leq 1 \tag{6}$$

where δ_{dem} and δ_{sup} are the demand and supply displacements respectively, β is the Park & Ang parameter (typically 0.10-0.15), F_y and δ_y are the yield force and yield displacement respectively and E_p is the cumulative plastic (or hysteretic) energy. For a system with nonlinear restoring force f_s, the plastic energy E_p is obtained by subtracting the elastic strain energy from the total strain energy

$$E_p = \int_0^t f_s \dot{u} dt - E_s \tag{7}$$

Considering that the quantity δ_{sup} is not uniquely defined because it depends on the specific characteristics of the structural system, equation (6) is recast in the following generalized form [10]:

$$D^*_{PA} = (\mu_{sup} - 1) \cdot D_{PA} = (\mu - 1) + \beta \frac{E_p}{F_y \delta_y}, \qquad \mu \geq 1, \; 0 \leq D^*_{PA} \leq (\mu_{sup} - 1) \tag{8}$$

where μ is the displacement ductility factor. It is worth noting that the denominator of the second term is related to the reduction factor q.

To form a common basis for the ensuing correlation studies, there is a need to adopt a relationship between q and T_μ/T_{el}. This is because the controlling structural parameters in the time-domain numerical studies are T_{el} and q but for the evaluation of spectral sustainability index the parameters are T_{el} and T_μ. The construction of an accurate relationship between the reduction factor q, the ductility ratio μ and elastic period T_{el}, has been the topic of much research effort and various approximate expressions can be found in the literature [e.g.15-16]. Here, the following expression [16] is employed because, despite its simplicity, provides reasonably accurate results

$$\begin{aligned} q &= (\mu - 1) \cdot \frac{T_{el}}{T_C} + 1 \qquad \text{for } T_{el} \leq T_C \\ q &= \mu \qquad\qquad\qquad\quad\; \text{for } T_{el} > T_C \end{aligned} \tag{9}$$

where T_C is the characteristic period of the ground motion.

3.1 Correlation studies

Firstly, the ability of sustainability index λ_2 to capture the damageability of individual records is demonstrated via the comparison between the computed response of an elastoplastic oscillator (T_{el} =0.5s, ξ = 5%) under the LEF-L and the scaled (to match spectral ordinates) A399-L records, for various levels of reduction factor q. Selected time history responses and the computed damage indices are shown in Figure 3.

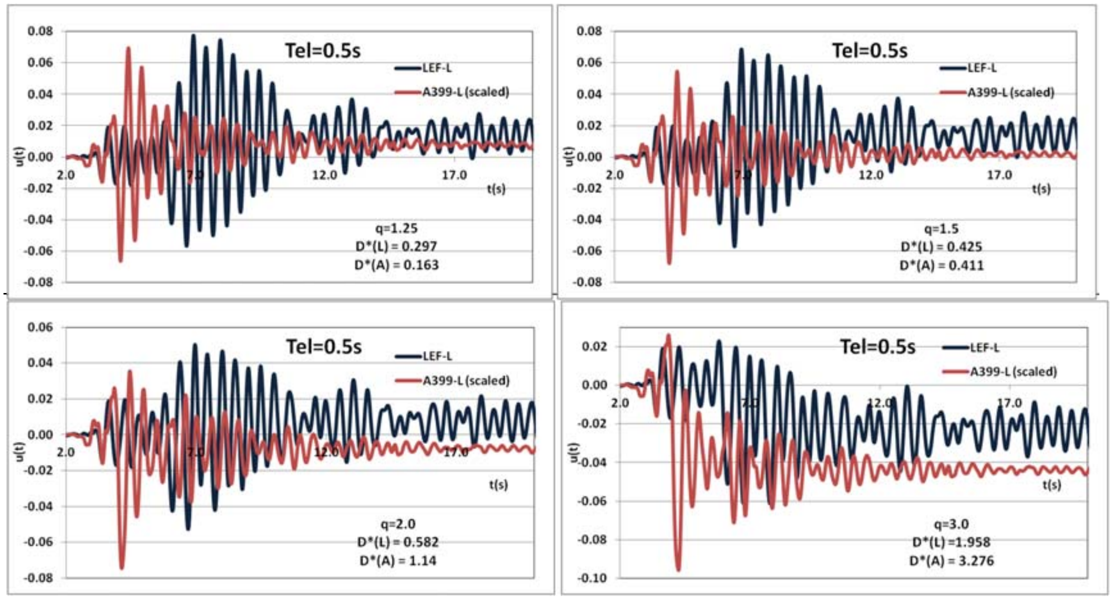


Figure 3. Response time histories and associated modified damage indices of an elasto-plastic sdof system under the LEF-L (blue) and scaled A399-L, records. From top-left: q = 1.25, 1.50, 2.0, 3.0

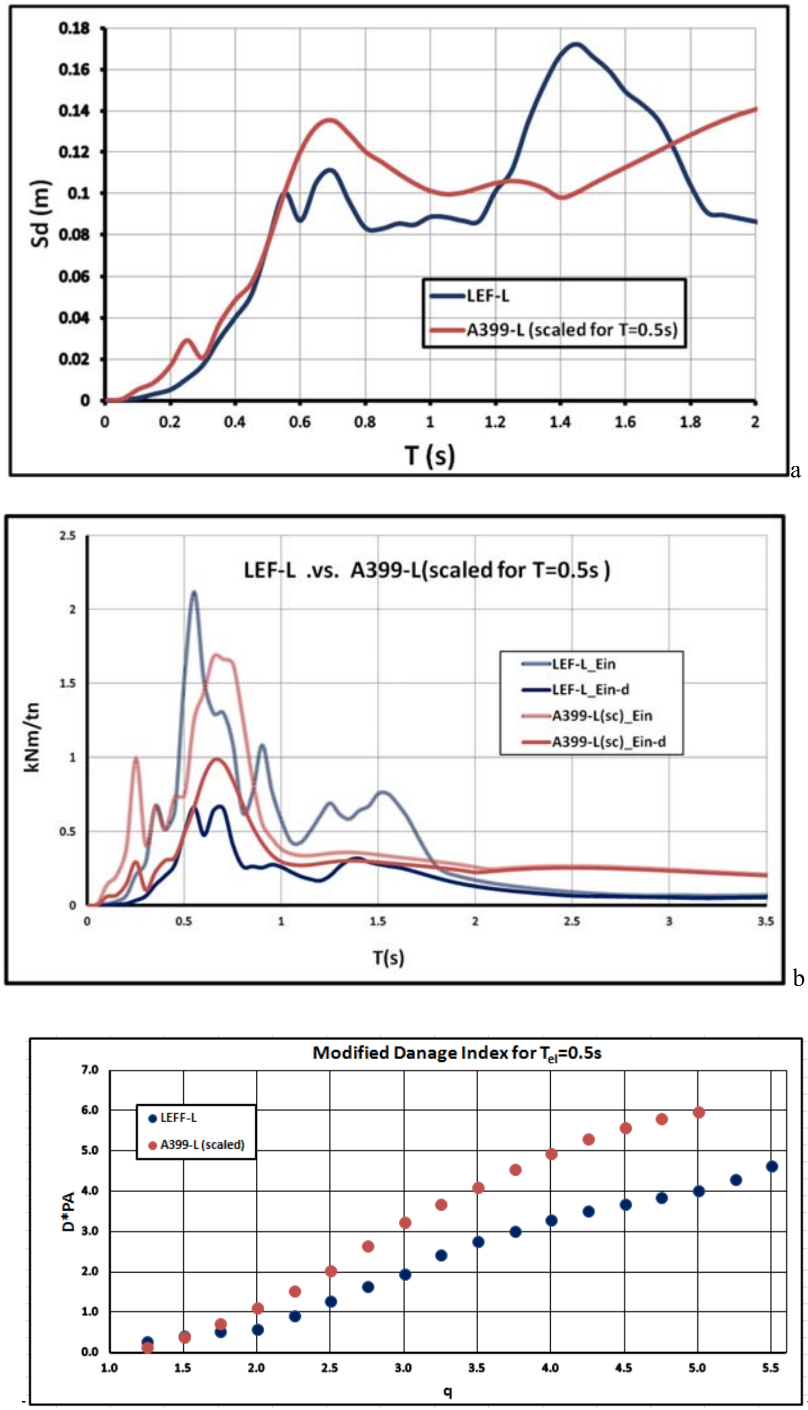
The above results are in full accordance with displacement spectra sustainability indices. It is observed that for small values of q (quasi-linear behavior) the LEF-L record displays greater damageability due to superior persistence of response peaks (higher values of displacement 'transversal' sustainability index λ_1), [10]. However, as q increases - corresponding to higher ductility factors μ and significant departure of the effective period Tμ from Tel - the situation is reversed. This was expected because the record A399-L exhibits higher values of displacement 'horizontal' sustainability index λ_2 for the whole range of period elongation ratio considered, as shown in Figures 2 and 4.

Strong correlation is also observed between energy sustainability measures and the computed damage. In Figure 4, the displacement response spectra alongside with the energy response spectra (for E_{in} and $E_{in\text{-}d}$ quantities) of the two records are compared. It is seen that despite the displacement spectral match (after scaling the A399-L), the energy content of the two records is quite different. In terms of input energy E_{in}, the LEF-L exhibits higher values for T ≤ 0.6s, corresponding to a small period elongation range ($T_\mu/T_{el} \leq 1.2$), while the scaled A399-L record ejects higher energy levels for 0.6s < T ≤ 0.85s, corresponding to an intermediate period elongation range $1.2 < T_\mu/T_{el} \leq 1.7$. Finally, LEF-L assumes greater input energy values for 0.85s < T ≤ 1.5s, corresponding to $1.7 < T_\mu/T_{el} \leq 3.0$, a fact that is not reflected in the computed damage.

It is interesting to note however, that the potentially damaging energy $E_{in\text{-}d} = E_{in} - E_d$ the A399-L exhibits higher energy values throughout the period elongation range $1.2 < T_\mu/T_{el} \leq 3.0$, in full accordance with the computed damage indices (also shown in Figure 4). Therefore, this energy quantity appears to be a more reliable and stable descriptor of damage potential and in what follows, the suitability and sufficiency of this energy term will be exclusively studied.

A more pertinent investigation for seismic risk applications, focuses on the correlation of the proposed index λ_{E2} with damage bounds computed from the whole set of records. This was firstly done by identifying the corresponding bounds of λ_{E2} index, separately for each of the seven structural periods examined. For example, the upper, median and lower bounds of the spectral sustainability index $\lambda_{E2}(T,\mu)$ of the potentially damaging energy $E_{in\text{-}d} = E_{in} - E_d$, for linear sdof (T=0.5s, ξ = 5%) under the action of the selected 12 strong motion records is shown in Figure 5.

Subsequently, the sustainability index bounds were compared with the corresponding bounds of the computed modified damage index D^*_{PA} (from the nonlinear dynamic analyses) and a very strong correlation was identified. As an example, the ratio of upper bound values of D^*_{PA} over the corresponding upper bound values of $\lambda_{E2}(T,\mu)$ for a sample of elastoplastic systems under the action of the whole set of records, is plotted in Figure 6. The translation from $\lambda_{E2}(T,\mu)$ to $\lambda_{E2}(T,q)$ is done via the implementation of equations (3) and (9) setting T_C = 0.5s in all cases of bounds estimation.



a

b

c

Figure 4. Comparisons of: (a) displacement response spectra S_d, (b) spectra of input energy E_{in} and potentially damaging energy E_{in-d}, (c) modified damage index D^*_{PA} for an elasto-plastic oscillator with elastic period T_{el} = 0.5s under the LEF-L and scaled A399-L, records

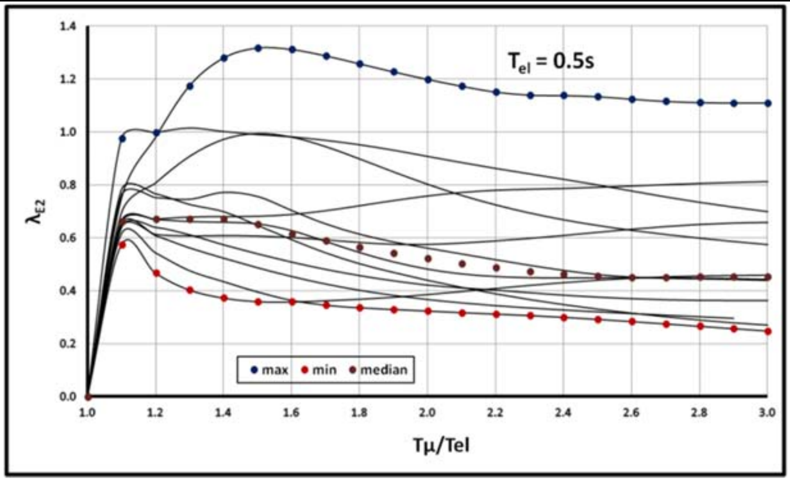


Figure 5. Bounds of energy (E_{in-d}) spectral sustainability index $\lambda_{E2}(T,\mu)$ for a linear sdof (T=0.5s, ξ = 5%) under the action of the selected 12 strong motion records.

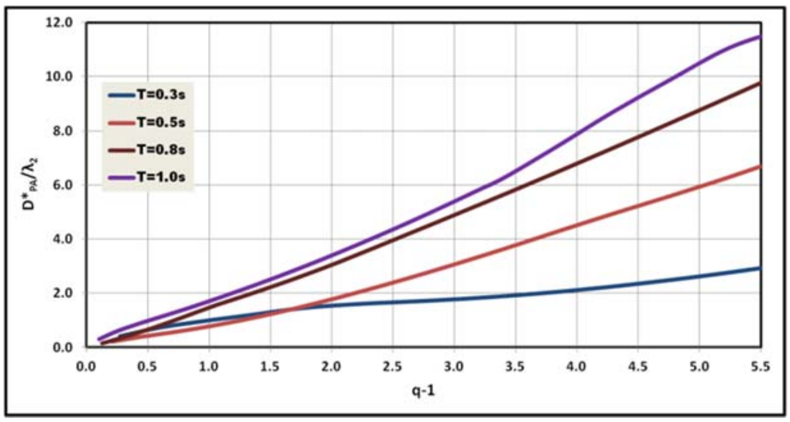


Figure 6. Correlation between the maximum values of energy (E_{in-d}) spectral sustainability index $\lambda_2(T,\mu)$ and the maximum computed values of the modified damage index.

4 DAMAGE BOUNDS PREDICTORS BASED ON ENERGY SPECTRAL SUSTAINABILITY

So far the ability of the proposed energy (E_{in-d}) spectral sustainability index $\lambda_{E2}(T,\mu)$ to capture the damage potential of strong motion records in a qualitative fashion, was explored and confirmed. In this section, a further step is taken towards the use of $\lambda_{E2}(T,\mu)$ for a quantitative prediction of damage. To this end, empirical relations are proposed, exploiting the aforementioned correlation studies between computed damage bounds and corresponding $\lambda_{E2}(T,\mu)$ bounds, shown in Figure 6.

The proposed damage empirical predictors have the following power form, for all bounds considered (upper, lower, median):

$$D^*_{PA}(\lambda_2, q) = \lambda_{E2} \cdot [a \cdot (q-1)^b] \tag{10}$$

where the coefficients a and b are given in Table 2 for a sample of structural periods

Comparisons between predicted damage bounds and computed modified damage indices from nonlinear dynamic analyses are presented in Figure 7. The predicted damage levels are in impressively good agreement with the computed results, manifesting the suitability of the proposed sustainability indices to form the basis of reliable damage predictors.

It should be noted that comparisons with q as an abscissa are only meaningful if the records are normalized to identical displacement spectral values at the elastic period of interest (as done herein). Otherwise, a given value of q corresponds to different structural systems. If unscaled records are utilized, the comparison should be made in terms of another quantity, such as V_y/W where V_y is the yield base shear and W is the weight of the structure.

	Upper bound		Lower bound		Median bound	
T(s)	a	b	a	b	a	b
0.3	1.25	0.65	0.60	1.20	1.05	1.49
0.5	0.85	1.20	1.05	1.31	1.10	1.21
0.8	1.50	1.15	1.19	1.40	1.32	1.18
1.0	1.98	0.97	1.03	1.46	1.33	1.23

Table 2 : Coefficients of empirical damage predictor

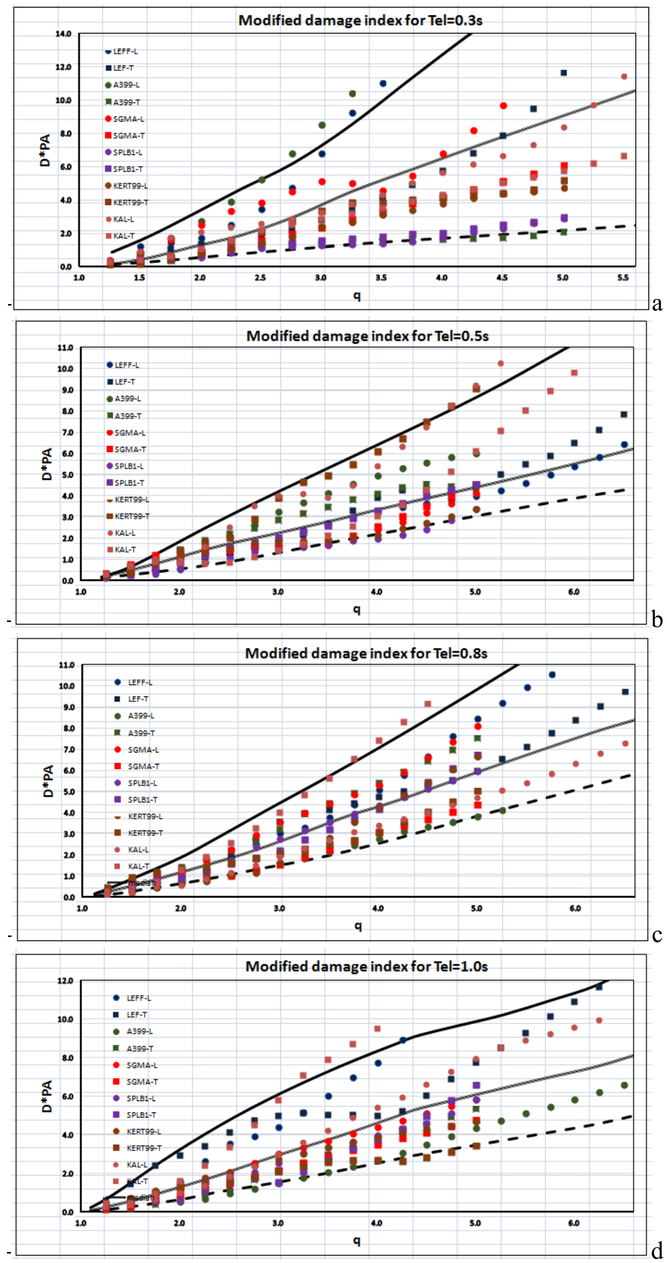


Figure 7. Predicted upper (solid line), lower (dashed line) and median (gray line) damage bounds vs computed modified damage indices (marks) of elastoplastic systems under the set of strong motion records.

5 CONCLUSIONS

In this work, a recently published methodology for damage potential characterization and quantification of earthquake records utilizing damage-related properties of 3-D displacement response spectra via the introduction of suitable spectral sustainability indices, is successfully extended for an alternative 2-D energy spectra implementation.

The results of parametric analyses, comprising elasto-plastic oscillators with seven elastic periods and a range of 20 reduction factors subjected to twelve selected strong ground motion records, confirm the strong correlation between the proposed energy (E_{in-d}) spectral sustainability index $\lambda_{E2}(T,\mu)$, with the observed response characteristics and the computed damage measures. Therefore, this index can be implemented for the selection of records with the greater damage potential among a set of normalized records sharing the same probability of occurrence (IM equivalence) and thus, facilitate the construction of fragility curve bounds.

Furthermore, the $\lambda_{E2}(T,\mu)$ index was utilized for the formation of simple yet reliable damage predictors with the ability to both follow the general trend of the nonlinear dynamic analysis and produce reasonably accurate bounds of well-established damage indices.

Admittedly, the validation of the aforementioned findings with the use of a substantially enriched sample of strong motion records is imperative. Furthermore, studies regarding the applicability of the proposed methodology for the damage prediction of more realistic structural systems are equally required.

ACKNOWLEDGEMENTS

This research has been co-financed by the European Union (European Social Fund–ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework, research project “Sei.V.A.S.: Seismic Vulnerability Assessment of the building stock in the city of Serres”, in ARCHIMED III Research Funding Program: "Strengthening Research Groups at the TEI of Serres" (mis 380873).

REFERENCES

[1] Ghisbain P. (2014), "*Damage-based Earthquake Engineering*", WIT Press.

[2] Katsanos EI, Sextos AG, Manolis GD. (2010), "Selection of earthquake ground motion records: a state-of-the-art review from a structural engineering perspective", *Soil Dynamics and Earthquake Engineering*, Vol. 30, pp.157–169.

[3] Baker J.W. (2013), "Conditional Mean Spectrum: Tool for ground motion selection", *Journal of Structural Engineering* (ASCE), Vol.137, pp.322-331.

[4] Luco N, Cornell CA. (2007), "Structure-specific scalar intensity measures for near-source and ordinary earthquake ground motions", *Earthquake Spectra*, Vol.23, pp.357–392.

[5] Baker J.W. (2013), "Trade-offs in ground motion selection techniques for collapse assessment of structures", *Vienna Congress on Recent Advances in Earthquake Engineering and Structural Dynamics*, 28-30 August 2013, Vienna, Austria, Paper No. 123.

[6] Chandramohan R., Lin T., Baker J.W., Deierlein G.G. (2013), "Influence of ground motion spectral shape and duration on seismic collapse risk." *10th International Conference on Urban Earthquake Engineering*, Tokyo, Japan, 9p

[7] Bojórquez E., Iervolino I. (2010), "A Spectral Shape-based Scalar Ground Motion Intensity Measure for Maximum and Cumulative Structural Demands", *Proceedings of the 14th European Conference on Earthquake Engineering*, 2010

[8] Bianchini M., Diotallevi P., Baker J.W. (2009), "Prediction of Inelastic Structural Response Using an Average of Spectral Accelerations", *10th International Conference on Structural Safety and Reliability* (ICOSSAR09), Osaka, Japan, 8p.

[9] Mousavi M., Ghafory-Ashtiany M., Azarbakht A. (2011), "A new indicator of elastic spectral shape for the reliable selection of ground motion records", *Earthquake Engineering and Structural Dynamics*, Vol.40, pp.1403-1416.

[10] Koliopoulos P.K. (2014), "Damage potential prediction of Greek earthquakes based on spectral sustainability measures derived from 3-D displacement response spectra: Part – I", *The Open Construction and Building Technology Journal*, Bentham Science Publishers Ltd., Vol.8, pp.34-45.

[11] Koliopoulos P.K., Margaris B.N. (2001), "The 1999 Athens (Greece) Earthquake: Energy and Duration – Related Response Spectral Characteristics of Different Site Conditions", *Proceedings of the 4th Int. Conf. on Recent Advances in Earthquake Engineering and Soil Dynamics*, No. 10.31.

[12] Kojima K., Fujita K., Takewaki I. (2014) "Building earthquake resilience in sustainable cities in terms of input energy", *Sustainable Cities and Society*, Vol.12, pp.46-62.

[13] Koliopoulos P. K., Kirtas E.A. (2002), "Plastic Energy Demand of the September 1999 Athens Earthquake", *Proceedings of the 12th European Conference on Earthquake Engineering*, CD ROM No. 220

[14] Koliopoulos P.K., Panagopoulos G.K., Mouratidis E.K. (2004), "Empirical Predictions of Plastic Energy Demand of Greek Earthquakes", *11th International Conference on Soil Dynamics & Earthquake Engineering*, 7-9 January 2004, University of California, Berkeley. (Paper No. 215)

[15] Karakostas C.Z, Athanasiadou C.J., Kappos A.J., Lekidis V.A. (2007), "Site-dependent design spectra and strength modification factors, based on records from Greece", *Soil Dynamics and Earthquake Engineering*, Vol. 27, pp. 1012-1027.

[16] Fajfar P. (2000), "A Nonlinear Analysis Method for Performance Based Seismic Design", *Earthquake Spectra*, Vol.16, pp.573-592.