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BIOMECHANICAL MODELLING AND SIMULATION OF SOFT TISSUES USING FRACTIONAL MEMRISTIVE ELEMENTS

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Abstract. *One of the main challenges in biomechanics is the modelling of soft tissues. The theory of fractional calculus is a well-adapted tool to the modelling of many physical phenomena, allowing the description to take into account some peculiarities that classical integer-order model simply neglect. Beside, memristive systems have a wide range of applications in modeling of the bioelectrical properties of human skin, human blood, storage, neural networks, chaotic systems and so on. In this work, we use the concept of fractional order memristive elements for biomehanical modeling of soft tissues, here bioelectrical properties of human skin as well as human blood. In literature, some models of human skin based on classical memristive approach are obtained but further improvements to the memristive models are possible where computational models are developed and presented. Further, we are interested in step input response for proposed Cole impedance models excited by a step current. Suitable numerical aproximations of inverse Laplace transform are used with respect to the simulation of the corresponding fractional (integro)-differential equations.*

Keywords: Soft tissue, Memristive elements, Fractional calculus, Circuit elements.

1 INTRODUCTION

The electrical property of any biological tissue depends on its intrinsic structure. Electrical admittivity of human tissue may be predominantly conductive or capacitive, or a combination of these, depending on tissue type, availability of charge carriers, frequency of the applied electric field, and is carried out by ions, dipoles or electrons, as well as holes (semiconductor) [1]. The relationship between the data of the applied stimulus and the response obtained as a function of frequency provides the impedance spectrum of tissues studied [2]. Transfer function analysis is a mathematical approach to relate an input signal (or excitation) and the system response. For an example, bioelectro-physical properties of human skin tissue, like most other soft tissues, exhibit electrical behavior [3,4]. To analyze skin impedance effectively, it is very desirable to introduce the skin impedance model. Electrical impedance studies in biological systems, including human skin, generally, relate to direct measurements of impedance and phase angle as functions of frequency, voltage, or current applied [4]. This research employs an electrical impedance model, which includes constant phase element (CPE).The Cole impedance model was postulated in its final form by Cole in 1940 [2]. This impedance model is based on replacing the ideal capacitor in the Debye model [3,4] for a general element (CPE). A special case of the general fractance device of fractional order is referred to as (CPE) which have shown numerous applications in the field of bioimpedance, which measures the passive electrical properties of biological materials, [4]. On the other hand, the theory of fractional calculus (FC) is a well-adapted tool to the modelling of many physical phenomena, allowing the description to take into account some peculiarities that classical integer-order model simply neglect. Application of FC in classical and modern physics greatly contributed to the analysis and our understanding of physico-chemical and bio-physical complex systems [5]. The importance of fractional order mathematical models is that they can be used to produce a more accurate description, and so give a deeper insight into the physical processes underlying long range memory behaviors. This property leads to simple (fractional) models, contrary to classical (integer) models that frequently require elaborated expressions. From mathematical point of view, the fractional integro-differential operators (FC) [5,6], are a generalization of integration and derivation to non-integer order (fractional) operators. Therefore, our understanding of biological systems organization requires FC as a mathematical tool [7,8]. A large number of useful biophysical studies reported applications of FC; however, they were limited to relatively small number of biological model system. Particularly, a memory function equation, scaling relationships and structural–fractal behavior of biomaterials and mathematical model based on fractional calculus, were used for the physical interpretation of the Cole-Cole (Cole) exponents [4]. So, three expressions for the impedance allow one to describe a wide range of experimental data: Cole–Cole function, Cole–Davidson function and Havriliak–Negami function [2,7,8].

Also, in this paper we suggest that the memristor is a necessary and useful building block modeling bioelectrical phenomena.We present the connection between FC (fractional order integral and derivative) and behavior of the memristive systems. As we will see, the fundamentals of FC are based on the memory property of the fractional order integral/derivative and therefore this connection is straightforward. Memristor is a new electrical element which has been predicted and described in 1971 by Leon O. Chua [9] and for the first time realized by HP laboratory in 2008. He proved that memristor behavior could not be duplicated by any circuit built using only the other three elements (resistor, capacitor, inductor), which is why the memristor is truly fundamental. In this work, we use the concept of fractional order memristive elements for biomehanical modelling of soft tissues, here bioelectrical properties of human skin as well as human blood. In literature, some models of human skin based on classical memristive approach are obtained but further improvements to the memristive models are possible. Also, a number of methods to extract the Cole impedance parameters have required measurement of either the impedance or frequency response of connected materials.Recently,in paper [10] a method of non-linear least squares fitting is applied to extract the single and double-dispersion Cole impedance parameters from collected current-excited step response datasets without requiring direct impedance measurements,[11]. In this paper, we proposed the human skin/blood structure as a more complex system where we have used fractional calculus and memristive approach to model bio-electrical impedance and applied derived models to describe bioimpedance properties of human skin as well as blood a test systems.Finally, we simulated step responses of three different dispersion Cole impedance models excited by a step current.

2 MEMRISTIVE SYSTEMS-FRACTIONAL APPROACH

2.1 Basic facts of memristive systems

Memristive systems are also used for modelling of biomechanical systems. Namely, memristor as nonlinear element was postulated by Chua in 1971 [9] by analyzing mathematical relations between pairs of fundamental circuit variables and realized by HP laboratory in 2008. Chua proved that memristor behavior could not be duplicated by any circuit built using only the other three elements (resistor, capacitor, inductor), which is why the memristor is truly fundamental.The four basic circuit variables, current *i,* voltage *v*, charge *q* and magnetic flux *φ*, give six different, possible combinations. The complete set of combinations is illustrated in Figure 1.

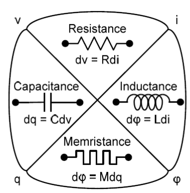


Figure 1. Connection of four basic electrical elements

Memristor is a contraction of memory resistor, because that is exactly its function: to remember its history. The memristor is a two-terminal device whose resistance depends on the magnitude and polarity of the voltage applied to it and the length of time that voltage has been applied. The missing element - the *memristor*, with memristance *M*-provides a functional relation between charge and flux, $d\varphi = Mdq$. However, the memristance is not necessarily linked to magnetic systems and can be regarded as the ratio between the time dependent voltage and current:

$$d\varphi = Mdq \Rightarrow vdt = Midt \Rightarrow M = v(t) / i(t) \ [\Omega]. \tag{1}$$

The memristor can be thought of as a variable resistor where the resistance is dependent on the amount of charge having passed the device in a given direction. A linear time-invariant memristor is simply a conventional resistor. In 1976 Kang and Chua [11] generalized the memristor concept to a larger class of nonlinear systems, called as ‘*memristive systems*’.

2.2 Fractional–order calculus definitions

Further, we present the connection between fractional calculus (fractional order integral and derivative) and behavior of the memristive systems. One important property of fractional operators is that they capture the history of all past events contrary to what occurs with integer derivatives that are ‘local’ operators. This means that fractional order systems have intrinsically a memory of the previous dynamical evolution. From mathematical point of view, the fractional integro-differential operators are a generalization of integration and derivation to non-integer order (fractional) operators. The most used definitions of a fractional derivative of order a are, respectively, the Riemann–Liouville, Grünwald-Letnikov and Caputo formulations:

$$ {}_{RL}D_{a,t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau)\,d\tau,\ t > a,\ \mathrm{Re}(\alpha) \in \left]n-1, n\right[\tag{2a} $$

$$ {}_{a}^{GL}D_t^{\alpha} f(\mathrm{t}) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{\left[\frac{t-a}{h}\right]} (-1)^k \binom{\alpha}{k} f(t-kh),\ \ t > a,\ \alpha > 0, \tag{2b} $$

$$ {}_{a}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_a^t (t-\tau)^{n-\alpha-1} f^{(n)}(\tau)\,d\tau,\ \ t > a,\ n-1 < \alpha < n \tag{2c} $$

The Caputo and Riemann-Liouville formulation coincide when the initial conditions are zero,[6]. For convenience, Laplace domain is usually used to describe the fractional integro-differential operation for solving engineering problems. For zero initial conditions [6], Laplace transform of fractional derivatives (Grunwald-Letnikov, Riemann-Liouville, and Caputo's), reduces to: $L\left\{ {}_0D_t^{\alpha} f(t) \right\} = s^{\alpha} F(s)$. In electrical systems was proposed the concept of fractional impedance, sometimes called *"fractor"*. The idea occurs interpolating between the three lumped electrical elements, namely the resistor, inductor and capacitor. Since these elements implement physically three consecutive (integer order) differential relations, we conceive the possibility of having other elements in between (i.e., of fractional order).

2.3 Fractional order memristive systems

Westerlund *et al.* [13] proposed a new linear capacitor model and described behavior of real inductor [14]. For a general input voltage $v(t)$ the current is

$$ i(t) = \mathrm{C}\left(d^{\alpha} v(t) / dt^{\alpha}\right) = C \cdot D_t^{\alpha} v(t) \tag{3} $$

where C is capacitance of the capacitor and α order is related to the losses of the capacitor. Also, for a general current in the inductor the voltage is

$$ v(t) = L\left(d^{\alpha} i(t) / dt^{\alpha}\right) = L \cdot D_t^{\alpha} i(t) \tag{4} $$

where L is inductance of the inductor and constant α is related to the *„ proximity effect"*. In that way, we can enlarge the family of elementary circuit elements, to fractional circuit elements in order to model many irregular and exotic nondifferentiable phenomena which are common and dominant to the nonlinear dynamics of many biological, molecular systems and nanodevices. Fractional circuit element can be characterized by a constitutive relation

$$ v^{\alpha}(t) = \mathrm{g}\left(i^{\beta}(t)\right) \tag{5} $$

where

$$ v^{\alpha}(t) = D^{\alpha} v(t),\ i^{\beta}(t) = D^{\alpha} i(t) \tag{6} $$

are any two complementary constitutive variables (i.e., current, charge, voltage, or flux) denoting input and output of the system and $D^{\alpha(\beta)}$ denotes fractional operator (fractional derivative or fractional integral). For example, in case of integer order systems ,when $\alpha = 0, \beta = 0$ and g is a linear function of i(t): $v(t) = R\ i(t)$, R is the resistance. The following are some other simple cases with integer *α, β*: resistor (*v, i*), ($\alpha = 0$, $\beta = 0$); inductor (*φ, i*), ($\alpha = -1, \beta = 0$); capacitor (*v, q*), ($\alpha = 0, \beta = -1$); memristor (*φ, q*), ($\alpha = -1, \beta = -1$); memcapacitor (*φ,* $i^{(-2)}$), ($\alpha = -1, \beta = -2$); meminductor ($v^{(-2)}$,*q*), ($\alpha = -2, \beta = -1$). For other (*α, β*) pair and more complex response functional *g*, there could be other very different devise. Moreover, the more convenient form of the current - voltage equation for the memristor is [15]

$$ M(q(t)) \int_0^t i(t)dt = \int_0^t v(t)dt \tag{7} $$

If $M(q(t))$ is a constant ($M(q(t)) \equiv R(t)$), then we obtain Ohm's law $R(t) = V(t)/I(t)$. If $M(q(t))$ is nontrivial, the equation is not equivalent because $q(t)$ and $M(q(t))$ will vary with time. Furthermore, the memristor is static if no current is applied. This is the essence of the *memory effect*, which allow us extending the notion of memristive systems to capacitive and inductive elements in the form of memcapacitors and meminductors whose properties depend on the state and history of the system [16, 17]. Similar to capacitor and inductor, the memristor is also not ideal circuit element and we can predict the fractional order model of such element. Applying the fractional calculus to relation (7), we obtain the following general formula for the fractional-order memristive systems:

$$M \cdot D_t^{-\alpha} i(t) = D_t^{-\alpha} \mathrm{v}(t)\,. \tag{8}$$

One can present memristive systems in general

$$K \cdot D_t^{\gamma} i(t) = D_t^{\beta} \mathrm{v}(t), \quad (\beta, \gamma \in R) \tag{9}$$

where K is the resistance, inductance, capacitance or memristance, respectively. Applying the Laplace transform technique to previous expression, we get the following relation for resulting impedance of the memristive system (MS) having properties equivalent to real electrical elements with fractional order mathematical models generally described as

$$Z_{MS}(s) = V(s) / I(s) = Ks^k,\ K \in R^+, \quad k = \gamma - \beta \in R \tag{10}$$

where k is the real order of the memristive system and for the ideal electrical elements has the following particular values, if:

$$k = 0,\ \gamma = 0, \beta = 0,\ \ K = R[\Omega],\ k = -1,\ \gamma = -1, \beta = 0,\ \ K = 1/C[F],\ k = 1,\ \gamma = 0, \beta = -1,\ \ K = L[H], \tag{11}$$
$$k = 0,\ \gamma = -1, \beta = -1,\ \ K = M(t)[\Omega]$$

We are able to define arbitrary real order k for the memristive system behavior description (10). The amplitude of this impedance function is $A = 20 \cdot k$ and the phase angle is $\varphi = k(\pi/2),\ k \in R$.Then the impedance of fractional capacitor is:

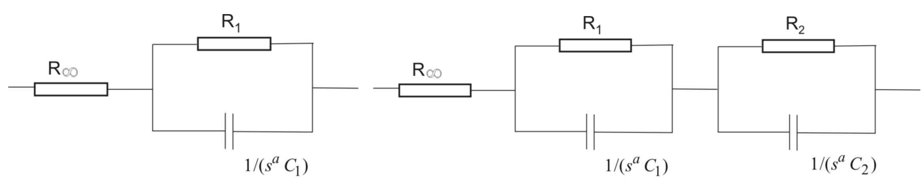
$$Z_{C_\alpha}(s) = Z_{CPE}(s) = \frac{1}{C_\alpha s^\alpha} = \frac{1}{C_\alpha \omega^\alpha} e^{j(-\alpha\pi/2)} \tag{12}$$

Electrical elements with such property are called *constant phase element* (CPE) for certain frequency range. In that way, the memcapacitor is believed to be useful within bioelectricity and neuroscience, potentially mimicking the widely used CPE.

3 SOME RESULTS RELATED TO COLE AND COLE-COLE EQUATION

In the field of bioimpedance measurements the Cole impedance model, is widely used for characterizing biological tissues and biochemical materials. The single-dispersion Cole model, shown in Fig. 2(a), is composed of three hypothetical circuit elements: high-frequency resistor R1, a resistor R1 and a CPE (C1, α_1). This model has become very popular because of its simplicity and good fit with measured data, illustrating the behaviour of impedance as a function of frequency. The impedance of the single-dispersion Cole model is then given by

$$\underline{Z}_\alpha(s) = R_\infty + \frac{R1}{1 + s^{\alpha_1} R_1 C_1} \tag{13}$$



a) b)

Figure 2a). Single-dispersion Cole impedance model, b) Double-dispersion Cole impedance model

An expanded model, the double-dispersion Cole model, is used to accurately represent the impedance over a larger frequency range or for more complex materials. This model, shown in Fig. 2b, is composed of an additional parallel combination of a resistor (R_2) and CPE in series with the single-dispersion Cole model with total impedance given by

$$\underline{Z}_\alpha(s) = R_\infty + \frac{R1}{1 + s^{\alpha_1} R_1 C_1} + \frac{R_2}{1 + s^{\alpha_2} R_2 C_2} \tag{14}$$

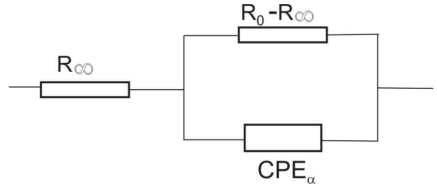


Figure 3. Equivalent circuit single-dispersion fractional Cole model according [8]

One of them, the Cole impedance model (for the specific electrical resistance) was introduced in its final form [4], by introducing *CPE*, with impedance $\underline{Z}_{CPE\alpha}(\omega)=1/\mathrm{C}_\alpha(j\omega)^\alpha$, or $\underline{Z}_{CPE\alpha}(s)=1/\mathrm{C}_\alpha s^\alpha$ in the *s*-domain. A complex impedance of the system is (Cole equation for single-dispersion model)

$$\underline{Z}_\alpha(\omega)=R_\infty+(R_0-R_\infty)\,\|\,\frac{1}{(s)^\alpha\cdot C_\alpha} \tag{15}$$

where the sign "||" denotes the parallel connection of complex resistance or

$$\underline{Z}_\alpha(s)=R_\infty+\frac{R_0-R_\infty}{1+(s\tau_\alpha)^\alpha}, \qquad \underline{Z}_\alpha(\omega)=R_\infty+\frac{R_0-R_\infty}{1+(j\omega\tau_\alpha)^\alpha}\bigg|_{s=j\omega} \tag{16}$$

as well as R_0 denotes a low-frequency resistor and R_∞ is a high-frequency resistor, $\alpha\in(0,1]$ is fractional *CPE* exponent-index. For $\alpha\to 0$, $\underline{Z}_\alpha(\omega)\to R_0$. In [8] equation for τ_α is (C_α is a fractional order capacitance) $\tau_\alpha=\sqrt[\alpha]{(R_0-R_\infty)\cdot C_\alpha}$ and this constant represents relaxation time constant**.** Recently, in paper [18] we introduced and proposed revisited continuous (distributed-order) Cole model as well as its approximation modified single-dispersion Cole model using modified distributed-order operator based on the Caputo-Weyl fractional derivatives,see Fig.3, The above approximation, in addition to defining the area where they should be fractional indices, in the range of high frequencies, may better describe the electrical properties of the system.

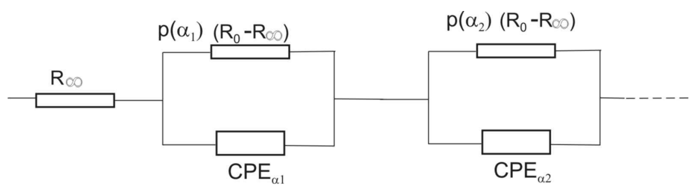


Figure 4. Electrical continuum model of the skin, based on the Cole equation

In that case, the skin is, in the electric sense, taken as serially continually many connected non-interactive, linear, reduced Cole elements p(α)·(R0 - R∞) ||Cα(j·ω)α and one R∞ (Fig. 4). Resistance p(α)·(R0 - R∞) characterized each individual reduced Cole element, where p(α) is a real non-negative function; $\tau_\alpha, 0<\alpha\leq 1$ are corresponding time relaxation constants, as a non-negative function of $\alpha:\tau_\alpha=\left(C_\alpha\cdot p(\alpha)\cdot(R_0-R_\infty)\right)^{1/\alpha}$. The equivalent total impedance $\underline{Z}_C(\omega)$ of this new electric circuit is given by the equation

$$\underline{Z}_C(\omega)=R_\infty+(R_0-R_\infty)\cdot\int_{0^+}^{1}\frac{p(\alpha)\cdot\mathrm{d}\alpha}{1+(\mathrm{j}\cdot\omega\cdot\tau_\alpha)^\alpha} \tag{17}$$

or, this expression (17) is the continuous generalization of the Cole equation. One of the main difficulties when using the distributed model is the large number and functionality of the distribution of material constants depending on the fractional index in relation to the number of experimental data. Here, we can obtain a discrete approximation of proposed Cole model as discrete series of reduced Cole elements and R_∞.

$$\underline{Z}(\omega)=R_\infty+(R_0-R_\infty)\sum_{i=1}^{n}\frac{p(\alpha_i)}{1+\left(\mathrm{j}\cdot\omega\cdot\tau_{\alpha_i}\right)^{\alpha_i}} \tag{18}$$

Equations for the two-dispersion Cole model (*n=2* in (18)) is

$$\underline{Z}_{\alpha_2}(\omega)=R_\infty+(R_0-R_\infty)\cdot\left(\frac{p(\alpha_1)}{1+\left(\mathrm{j}\cdot\omega\cdot\tau_{\alpha_1}\right)^{\alpha_1}}+\frac{p(\alpha_2)}{1+\left(j\cdot\omega\cdot\tau_{\alpha_2}\right)^{\alpha_2}}\right),\ p(\alpha_1)+p(\alpha_2)=1,\ 0<\alpha_1,\alpha_2\leq 1, \tag{19}$$

4 EXAMPLE OF FRACTIONAL ORDER OF MEMRISTIVE SYSTEM: HUMAN SKIN

Fundamental electrical mechanisms behind many bioelectrical phenomena may therefore involve significant memristive effects. As far as the biomedical field is concerned, there are a many observed hysteretic or nonlinear i-v characteristics are found in the field of bioimpedance and its allied fields [17,19,20] that may potentially fit better in a memristive framework than in any other. The memristor shows many interesting features when describing electrical phenomena, especially at small (molecular or cellular) scales and can in particular be useful for bioimpedance and bioelectricity modeling. In paper [20] authors presented model for sweat duct conductivity based on memristor theory. Current responses of human sweat capillaries are shown to behave as memristive systems since the resistance of the capillaries depends on the current history, i.e. behaving memristively. Our skin behaves as a memristor due to the dominance of the sweat ducts on the skin conductance at sufficiently low frequencies [9,20]. The equivalent electrical model of the human skin may therefore involve the memristor as suggested in Fig. 5. The new thing in this manner is that the traditional resistor for sweat conductance is substituted with the memristor of fractional order in contrary to of memristor of integer order,suggested in [19]. In human skin, the memristance property will be vanished at high frequency.

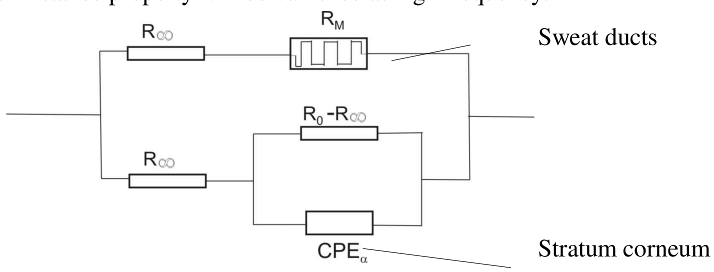


Figure 5. Equivalent electrical model of human skin with memristor of fractional order

The primary property of the memristor is the memory of the charge that has passed through it, reflected in its effective resistance M(q), $v(t) = M(q))i(t)$ .This is a generalization of the memristor concept where the memristance is controlled by any number of additional state-variables (in our case just one,$x(t)$), which may also be coupled to each other in complicated nonlinear ways. Thus, memristive systems should be rich enough to capture some of the nonlinearities that are ubiquitous in bioelectrics. In paper [19] authors showed that electro/osmosis in human sweat ducts are memristive nature as follows:

$$M = R\left(\tau + (1-\tau)\,x\right) \approx -R\tau x, \qquad \tau \gg 1\,, \tag{20}$$

where $R = \rho D / A,\ \tau = r / 2d,\ \ x = w / D,\ A = \pi r^2$, ρ resistivity of the fluid as well as the geometry of the duct $D, r, A, d, A = \pi r^2$, D is the total length of the duct, and $a \approx 2\pi r d, \quad d \ll r$ denoting the thickness of the film. Also, it is fullfilled that

$$\frac{dx}{dt} = k\frac{dq}{dt} = ki(t), \quad k = \beta\rho / V \tag{21}$$

Here, in this paper we introduce the fractional differential equation of memristor element and it can be given by

$$\frac{d^{\alpha}x}{dt^{\alpha}} = ki(t), \tag{22}$$

By differentiating both sides of (22) and combining it yields:

$$\frac{d^{\alpha}M}{dt^{\alpha}} = -R\tau\frac{d^{\alpha}x}{dt^{\alpha}} = -R\tau k i(t) = -\lambda i(t) \tag{23}$$

Finally, one can determine $M(t)$:

$$M(t) = -\lambda D_t^{-\alpha} i(t)\,. \tag{24}$$

Taking into account $v(t) = M(q))i(t)$ and procedure of fractional integration [21] memristor resistance as a function of the input voltage can be obtained by:

$$R_M^{\alpha+1} = M^{\alpha+1} = M_{in}^{\alpha+1} - \alpha(\alpha+1)\lambda \int_0^t (t-s)^{\alpha-1} v(s) ds \tag{25}$$

The equivalent total impedance $Z_{HSmem}(s)$ of this new electric circuit (Fig.5) is given by

$$\underline{Z}_{HS_{mem}}(s) = \left(R_\infty + R_M(s)\right) \| \left(R_\infty + (R_0 - R_\infty) \| CPE_\alpha(s)\right) \tag{26}$$

or

$$\underline{Z}_{HS_{mem}}(s) = \frac{\left(R_\infty + R_M(s)\right)\left(R_0 + R_\infty (s\tau_\alpha)^\alpha\right)}{\left(R_0 + R_\infty + R_M(s)\right) + \left(2R_\infty + R_M\right)(s\tau_\alpha)^\alpha} \tag{27}$$

On the other side, in study [22] authors observe that, the human blood also possess the same characteristics as like memristor. In the vein of blood, skin also possesses hysteresis like characteristics at very low frequency and it is vanish at high frequency. Equivalent electrical cirucit for blood tissue, erythrocytes, leukocytes and plasma is similar as Fig. 1a) with R_S, R_P, C_P so one can obtain

$$\underline{Z}_{blood}(s) = R_S + \left(R_P \| C_P s^\alpha\right) = R_S + \frac{R_P}{1 + s^\alpha R_p C_p} \tag{28}$$

which include memcapacitor of fractional order, $1/C_P s^\alpha$.On the basis of Cole's proposal to add a degree of extra freedom to solve the RC circuit for characterization purposes and to improve the correlation in the adjustment to experimental data, they obtained α, R_s, R_P, C_P as follows:

$$\begin{aligned} &\text{erythrocytes } \alpha \to 0.975,\ R_S = 673.7\,\Omega,\ C_P = 2.29\cdot 10^{-8}\ F,\ R_P = 120710\ \Omega, \\ &\text{leukocytes } \alpha \to 0.99,\ R_S = 475.6\,\Omega,\ C_P = 2.188\cdot 10^{-8}\ F,\ R_P = 341550\ \Omega, \\ &\text{plasma } \alpha \to 0.99,\ R_S = 496.5\,\Omega,\ C_P = 2.1517\cdot 10^{-8}\ F,\ R_P = 273750\ \Omega, \end{aligned} \tag{29}$$

5 NUMERICAL EXAMPLES

In fractional order systems we have two transfer functions of particular interest, the Cole–Cole and Davidson–Cole and Havriliak–Negami (HN) models [23] given respectively by:

$$\underline{Z}_C(s) = K / 1 + \left(\frac{s}{\omega_0}\right)^\alpha \quad \underline{Z}_{D-C}(s) = \frac{K}{\left(1 + \left(\frac{s}{\omega_0}\right)\right)^\gamma}, \underline{Z}_{H-N}(s) = \frac{K}{\left(1 + \left(\frac{s}{\omega_0}\right)^\alpha\right)^\gamma} \tag{30}$$

where $\alpha, \gamma \in R$, $K, \omega_0 \in R^+$ and s denotes the Laplace variable. Also, we using $L\{..\}$ the Laplace operator and t time, we have known the relationships:

$$L\left\{\omega_0^\alpha t^{\alpha-1} E_{\alpha,\alpha}\left[-(\omega_0 t)^\alpha\right]\right\} = 1 / \left(1 + \left(\frac{s}{\omega_0}\right)^\alpha\right), \quad L\left\{\omega_0^\gamma e^{-\omega_0 t} \frac{t^{\gamma-1}}{\Gamma(\gamma)}\right\} = 1 / \left(1 + \left(\frac{s}{\omega_0}\right)\right)^\gamma, \tag{31}$$

$$L\left\{\omega_0^{\alpha\gamma} t^{\alpha\gamma-1} E_{\alpha,\alpha\gamma}^\gamma\left[-(\omega_0 t)^\alpha\right]\right\} = 1 / \left(1 + \left(\frac{s}{\omega_0}\right)^\alpha\right)^\gamma$$

where $\Gamma(z), z \in \mathbb{C}$, is the Gamma function, and $E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$ and $E_{\alpha,\beta}^\gamma(z) = \frac{1}{\Gamma(\gamma)} \sum_{k=0}^{\infty} \frac{\Gamma(\gamma + k) z^k}{k!\Gamma(\alpha k + \beta)}$ represent the two and three parameters Mittag-Leffler functions, respectively.

5.1 Excitation Signals Response

Beside, due to complexitity of transfer functions,see (27) we are interested here to obtain impulse and particularly step input response. Also, in paper [10] it is applied a suitable method to extract the single and double-dispersion Cole impedance parameters from collected current-excited step response datasets without requiring direct impedance measurements. In this section, the response of fractional order different memristive systems under different input current-step signals is obtained and presented.

a) We consider a case of erythrocytes $\alpha \to 0.975,\ R_S = 673.7\,\Omega,\ C_P = 2.29\cdot 10^{-8}\ F,\ R_P = 120710\ \ \Omega$ with Cole impedance model (28) where the s-domain expression of the output voltage of (28) induced by a current-step of amplitude I_0

$$v(s) = I(s)\underline{Z}_{eryt}(s) = \frac{I_0}{s}\left(R_S + \frac{R_P}{1 + s^{\alpha} R_p C_p}\right) \tag{32}$$

The time domain expression of the output voltage $v(t)$ is calculated numerically, (see Fig.6) using a specific class of techniques for the numerical inversion of Laplace transforms (NILT), (33) namely based on a complex Fourier series approximation, and connected with a quotient-difference algorithm to accelerate the convergence of infinite series arising in the approximate formulae, [24]:

$$\tilde{f}(t) = \exp(ct^T)\left(\prod_{i=1}^{n} \tau_i^{-1}\right)\sum_{m_1=-\infty}^{\infty} \ldots \sum_{m_n}^{\infty} F(s)\exp\left(j\sum_{i=1}^{n} m_i \Omega_i t_i\right),\ \ \omega_i = m_i \Omega_i,\ \ \Omega_i = 2\pi / \tau_i \tag{33}$$

On Fig. 6 it is presented influence of parameter C_p for three values:

$$C_{P1} = 2.29\cdot 10^{-8}\ F, C_{P2} = 2.29\cdot 10^{-7}\ F, C_{P3} = 1.15\cdot 10^{-6}\ F,$$

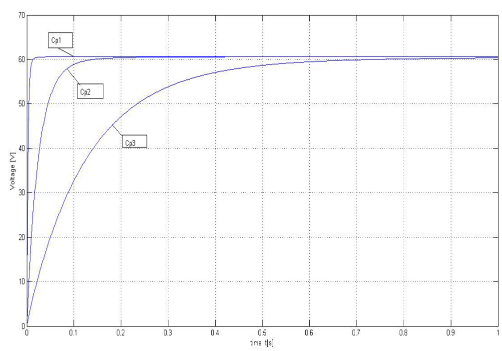


Figure 6. Simulated step responses of (a) single dispersion Cole impedance models excited by a step current of 0.1 mA $\alpha \to 0.975,\ R_S = 673.7\,\Omega,\ C_P = 2.29\cdot 10^{-8}\ F,\ R_P = 120710\ \ \Omega$

b) We consider a case of discrete approximation of proposed electrical continuum Cole model *n=2*, using (19) for $p_1 = p = 0.1, 0.5, 0.95$

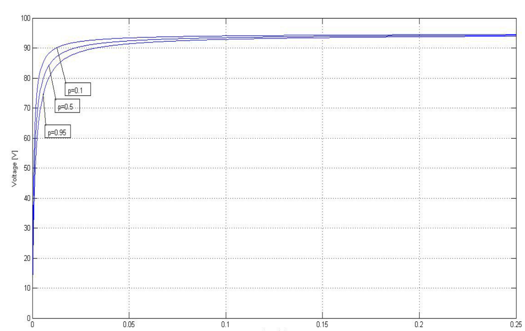


Figure 7. Simulated step responses of discrete approximation Cole impedance model excited by a step current of 1 mA, $R_\infty = 0.17k\Omega,\ R_0 = 95k\Omega, \tau_{\alpha_1} = 2.2710^{-3}, \tau_{\alpha_2} = 0.88510^{-3}, \alpha_1 = 0.885, \alpha_2 = 0.733$

c)Finally, we present step responses for equivalent electrical model of human skin with memristor (for *Rm* =const $R_m = 10, 30, 50\ \Omega$)

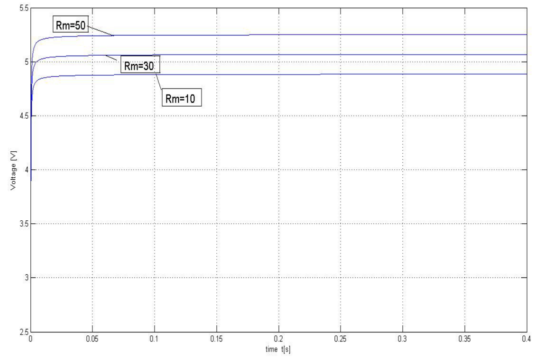


Figure 8. Simulated step responses of (27) model excited by a step current of 10 mA, $R_\infty = 500\Omega,\ R_0 = 11.7k\Omega, \tau = 510^{-3}, \alpha = 0.72$

6 DISSCUSIONS

In this paper we applied the concept of fractional order memristive elements for biomehanical modeling of soft tissues, specially human skin as well as human blood. We presented equivalent electrical model of human skin discrete approximation continuum Cole impedance model as well as expended electrical model of human skin which include influence of sweat ducts. Recently, it is shown that using suitable fitting method one can extract the corresponding Cole impedance parameters from collected current-excited step response datasets without requiring direct impedance measurements. Finally, we simulated step responses of three different dispersion Cole impedance models excited by a step current. Suitable numerical aproximations of inverse Laplace transform are used which are based on a complex Fourier series approximation.

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