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OPTIMAL CONTROL OF VORTEX SHEDDING USING DETAILED MODELS

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Abstract. *Vortex shedding is a naturally occurring phenomenon that often needs to be controlled. In applications where the flow-structure interaction may damage the structure, vortex shedding needs to be minimized, actively or passively. On the other hand, in flow energy harnessing applications, it is beneficial for vortex shedding to be maximized. Here, we formulate vortex shedding control as a dynamic optimization problem. We are concerned with wake formation behind a cylinder, for laminar flow with Reynolds number of 100. We consider that the angular speed of the cylinder can be controlled freely within certain limits. Our objective is to minimize vortex shedding which would minimize the lift coefficient. The latter is the objective function for our optimal control problem. Since wake formation is periodic, we assume that the optimal solution would also be periodic and could be expressed as a sinusoidal expression with degrees of freedom that are determined by the mathematical programming solution. Following our approach, the temporal profile for the cylinder angular speed is calculated offline and is considered set for a given Reynolds number. We consider two cases using 2 and 3 independent variables expressing more and less constrained cases and obtain efficient controllers for both cases.*

1 INTRODUCTION

Flow separation is commonly exhibited in nature and the built environment. This phenomenon gives rise to vortex shedding that can potentially damage the structure behind which vortices are formed, due to the induced vibrations. Hence, it is important that this phenomenon is controlled effectively. Both active and passive control techniques can be applied to this class of problems. This work work concerns the development of an active optimal control strategy.

Boundary layer control by means of rotating cylinders is one of the most promising among the proposed techniques and is the one followed here [1]. The proposed method is based on a detailed model of a system exhibiting laminar vortex shedding. Namely, a fluid stream of uniform velocity is directed towards a rotating cylinder causing the flow to separate. This system is simulated using the commercial Computational Fluid Dynamics package COMSOL Multiphysics [2]. The 2D model used here is transient and has been presented and validated in recent studies [3-4]. It comprises of spatially discretized Navier-Stokes and continuity equations and results in the computation of velocity and pressure profiles for a given fluid inlet velocity and cylinder rotation. This model is treated as a black-box tool for optimization purposes.

An optimal control problem is formulated, which calculates the optimal cylinder rotation strategy for a given Reynolds number using the detailed transient model as a constraint. The dynamic optimization problem at hand is solved using feasible path method and the implementation can be seen as a variant of single-shooting [5]. It is considered that the cylinder angular velocity follows a sinusoidal law in time, the parameters of which (amplitude and frequency) are the independent variables of the optimization problem. The objective of the latter is the minimization of vortex shedding, which is expressed using a suitable measure.

From a control perspective, the control variable is the lift coefficient, which is used as a measure of the vortex shedding magnitude. The manipulated variables are the scalars which parameterize the sinusoidal control law for the cylinder rotation and the disturbance is the Reynolds number. Here we consider the Reynolds number to be constant. We show that this formulation significantly decreases wake formation in comparison to the uncontrolled case of stationary cylinder.

The novelty of this contribution mainly lies in the formulation of the optimal control problem. Alternatively to rotating the cylinder, one may choose to control the system at hand by suction and blowing [7]. In [1], a

Proper Orthogonal Decomposition model reduction-based approach is used for optimal control, in which the modes are used as state variables for control. In [7], an adjoint method is used to control the flow so that the amplitude of the variation of the drag coefficient is reduced. Feedback control has also been used in vortex shedding systems [8].

The rest of this contribution is organized as follows: in Section 2 we present the basis for the model on which the optimization study relies. Section 3 presents the formulation of the optimization problems that concern us. Section 4 illustrates the optimization results, contains a discussion on the results and on the optimizer implementation. Finally, this contribution ends with some concluding results and a discussion of possible extensions of this work.

2 DETAILED MODELLING OF A VORTEX SHEDDING SYSTEM

We consider the two-dimensional standard cylinder flow benchmark problem, of a fluid of density ρ, past a cylinder of diameter D [1]. The governing set of PDEs is comprised of the Navier-Stokes equations and the continuity equation. In dimensionless form, these are:

$$\begin{aligned} &\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 u \\ &\nabla \cdot u = 0 \end{aligned} \tag{1}$$

The Reynolds number is based on the inflow velocity. The boundary conditions for (1) are: $u = (1, 0)\ on\ \Gamma_1$, $p = 0\ on\ \Gamma_3$, $\frac{\partial u_x}{\partial y} = 0,\ u_y = 0\ on\ \Gamma_2, \Gamma_4$ and $u = (\gamma_x, \gamma_y)\ on\ \Gamma_c$, γ being the angular velocity of the cylinder. The initial conditions used were: $u|_{t=0} = (0, 0)$. The equations were discretized on Finite Elements using Lagrange P2P1 elements, resulting in 4528 elements and 20766 degrees of freedom. The computational domain for the benchmark study, as well as its dimensions, is shown in Fig. 1 (left).

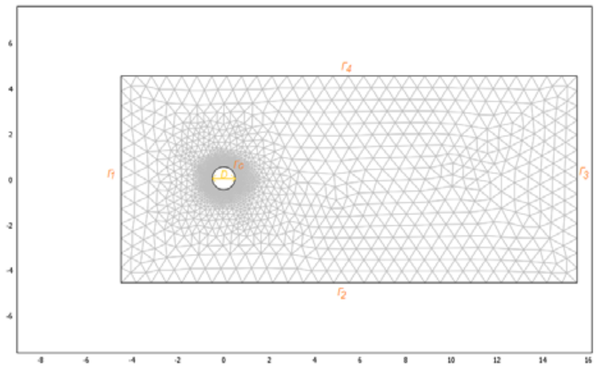


Figure 1: The computational domain for the benchmark problem and the discretization mesh.

The problem was formulated and solved using COMSOL Multiphysics [2] and validated against literature data [1]. The discretized system of the set of Equations (1) and the corresponding boundary conditions will hereon forth be mentioned as $f_{NS\text{-}c} = 0$. Every solution of this Equation comprises a set of velocity and pressure profiles for a given time that satisfy the physics of the problem, expressed by Equations (1).

The two parameters of the system in (1) is Reynolds number (*Re*) and the angular velocity of the cylinder (γ). As a benchmark, the laminar case of Re=100 and γ=0 was simulated, that is known to exhibit vortex shedding and turns to periodic. The lift and drag coefficients of the cylinder can be expressed using the *x*- and *y*-components of the force acting upon the cylinder and are respectively:

$$C_L = 2F_y,\ C_D = 2F_x \tag{2}$$

Where $F = (F_x, F_y) = \oint_{\Gamma_c} \left(p\hat{n} - \frac{1}{\mathrm{Re}} \frac{\partial u}{\partial n} \right) d\Gamma$, where $\hat{n}$ is the unitary vector perpendicular to the surface. Figure 2 shows the time evolution dynamic of these coefficients, from where a period of 5.5 time units is found to characterize the oscillations of the flow system.

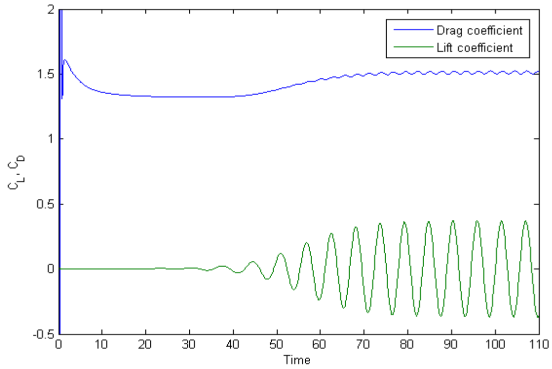


Figure 2: Time evolution of the lift and drag coefficients computed with a full-scale CFD model.

3 OPTIMAL CONTROL

3.1 Base case for optimization

The objective of this work is to minimize wake formation in the system at hand and we do so by minimizing the lift coefficient of Equation (2). The independent variable of the system (1) is the angular velocity γ. Since wake formation is periodic as we illustrated in the previous section, we consider using a periodic expression for our independent variable. Specifically, we parameterize γ using a sinusoidal expression of time and employing three independent variables $x = [x_1 \;\; x_2 \;\; x_3]^T$ as follows:

$$\gamma(x,t) = x_1 \sin(x_2 t + x_3) \tag{3}$$

The formulation of Equation (3) allows us to find the optimal magnitude, frequency and phase for the sinusoidal control action so that the vertical force applied to the cylinder is minimal.

The time horizon for all optimization problem, which we formulate, is equal to the period of the phenomenon for the case of stationary cylinder, T. T=100 for the benchmark case of Re=100. Applying a control action to the system perturbs and alters its periodicity. Further work will determine the time horizon that needs to be used in these problems. An approach would be to wait for the controlled system to reach periodic state and then start calculating the objective function; however, there is no guarantee that the controlled system will reach periodic state, or that the characteristics of this state would not depend heavily on the values of the independent variables, hence hindering the objective calculation.

For all the optimization problems that are examined here, we consider that initially the cylinder is stationary and the possibility of control is only feasible after the system has reached a periodic state. We simulate the system with γ = 0 with zero initial conditions as mentioned in Section 2 for adequate time, when the system reaches steady state. The time instance when we begin controlling the system is t_{in}. Here we chose t_{in}=100, as at this time it has been shown in Figure 2 that we have reached periodic state. The initial conditions (velocity and pressure profiles) for the optimal control problems formulated are the state variables for γ=0 and t=t_{in}.

We formulate a dynamic optimization problem to identify the values of x that minimize the

$$\begin{aligned} x &= \arg\min_x \left(\int_{t_{in}}^{t_{in}+T} \left(C_L(\gamma(x,t),u,t) \right)^2 dt \right) \\ s.t. \quad & \gamma(x,t) = x_1 \sin(x_2 t + x_3) \\ & f_{NS-c}(\gamma(x,t),u,t) = 0 \end{aligned} \tag{4}$$

3.2 Optimization with alternative objectives

We also consider the more constrained case, where the the peak amplitude of the sinusoidal expression (3) is equal to one. In this case, Equation (3) lacks the term x_1 and the optimization problem is:

$$
\begin{aligned}
& x = \arg\min_{x} \left(f(x,u,t) \right) \\
& s.t. \quad \gamma(x,t) = \sin(x_1 t + x_2) \\
& \qquad f_{NS-c}(\gamma(x,t),u,t) = 0
\end{aligned}
\tag{5}
$$

Since the number of independent variables in this case is less than the previous (2 versus 3) and therefore the computational cost is less, we use this case to examine different scenarios. One scenario is to minimize the downforce applied to the cylinder, by minimizing the lift coefficient throughout the control horizon. This is expressed as:

$$f_B(x,u,t) = \int_{t_{in}}^{t_{in}+T} \left(C_L(\gamma(x,t),u,t) \right) dt \tag{6}$$

An alternative objective is to minimize the distance between the lift coefficient and a given target. This is a setpoint tracking (servo) problem. This is expressed as:

$$f_A(x,u,t) = \int_{t_{in}}^{t_{in}+T} \left[C_L(\gamma(x,t),u,t) - C_{L,t\arg et} \right]^2 dt \tag{7}$$

This formulation would also be employed for a disturbance rejection problem. Generally speaking $C_{L,t\arg et}$ need not be constant with time. This will be discussed more in detail in the next Section.

4 RESULTS

This section presents optimization results for the 2 and 3 independent variable cases presented in the previous section. The optimization problem has been solved using MATLAB's active-set implementation of active-set deterministic optimization [5].

The bounds for the independent variables are chosen so that:

1. The magnitude of the control action x_1 is set to a design value that is greater than 0. This is because if x_1=0, for all x_2 and x_3, the value of f is the same, since the cylinder would be stationary. Therefore a plateau is formed which would compromise the optimization problem if the latter was solved with a gradient-based technique. We need the minimum acceptable value of x_1 to be far enough from 0 to avoid the attractor of the trivial solution of stationary cylinder.
2. The frequency of the sinusoidal control action x_2 is left to vary between $-\pi/2$ and $\pi/2$ as this is the period of a sinusoidal function.
3. The phase of the sinusoidal control action x_3 is left to vary between 0 and $T/2$ due to the periodicity of the function.

The computational cost of the optimization procedure is nontrivial. The computational time depends on the strategy used for optimization: deterministic or stochastic, local or global, feasible or infeasible path, i.e. whether the constraints (the underlying CFD problem) is solved in series with the optimizer iterations or simultaneously with optimization correspondingly. In the former case, which is the one followed here, all intermediate optimization points are guaranteed to satisfy the physics of the problem, whereas in the infeasible case only the optimization result will be guaranteed to satisfy all constraints. In such problems it is generally beneficial to employ commercial, or optimized in-house developed CFD codes for the solution of the (equality) constraints, as these tend to include implementations of a set of specialized methods that facilitate convergence. Generally speaking, gradient-based optimizers employ deterministic methods based on Newton-Raphson method, which may prove inefficient or even unable to solve the Navier-Stokes and continuity equations.

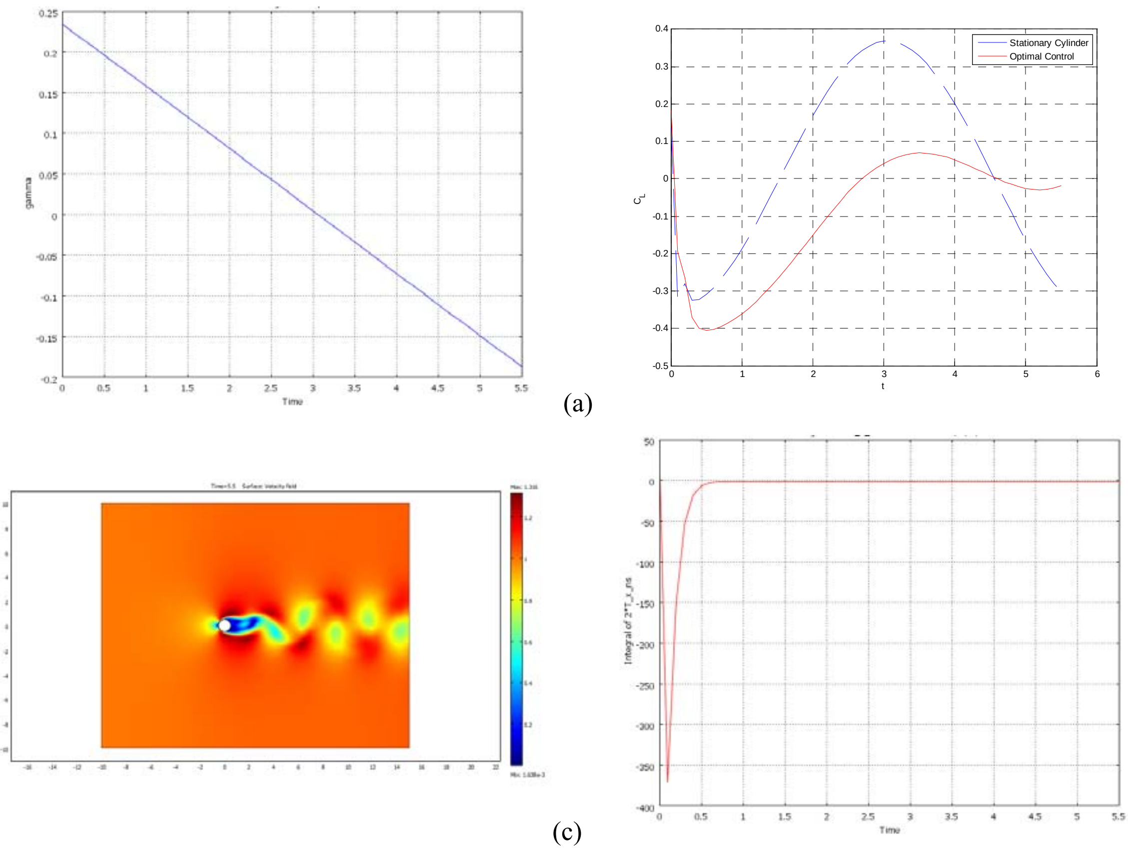
In the following subsections we will refer to function evaluations rather than actual computational time. Each function evaluation roughly translates to 5 minutes of CPU time for our CFD problem setup on the computational time we had available. Most of the computational time is expended on the solution of the constraints (i.e. the CFD problem). This need not be the case if a different strategy, or problem formulation was employed (e.g. if an infeasible path algorithm was used, as mentioned above).

The results that will be presented in this section are local. Global optimization methods typically employ more function evaluations and therefore more computational time. Both stochastic (e.g. genetic algorithms, simulated annealing etc.) and gradient-based global optimization can be applied to this optimization problem. A special class of deterministic global optimization techniques is multi-start that examines a larger part of the parametric space than local optimization techniques, but provide no guarantee of global optimality. Further work will examine the application of such techniques to the optimization problems formulated here.

4.1 3-degree of freedom optimal control

The optimal control problem solved here is the one presented in Equation (4). The objective is to minimize the magnitude of the lift coefficient at the time interval of duration T following enabling the controller, where T is the period of the wake formation for the case of stationary cylinder, subject to the Navier-Stokes and continuity equations being satisfied. The bounds for the independent variables are: $\begin{bmatrix} 0.5 \\ -\pi/2 \\ 0 \end{bmatrix} \le x \le \begin{bmatrix} 10 \\ \pi/2 \\ T/2 \end{bmatrix}$.

The optimal solution identified is $[1.1450 \quad -0.0673 \quad 0.2057]^T$, for which the value of f is 7.6279. The objective function value for the stationary cylinder is 11.8922, which is 55.90% higher than the optimal. The number of function evaluations (CFD model calls) is 150. The optimization results are presented in Figure 3.



(a) (b) (c) (d)

Figure 3: Optimization results for the 3-dof case: optimal control action (a), resulting lift coefficient (b), velocity field profile after 5.5 time units (c) and temporal profile of the corresponding drag coefficient (d).

One may observe that the optimization results in a quasi-linear profile for the control action (Figure 3a). Actually this action is sinusoidal, following Equation (3). As we can see in Figures 3b and 3d, the periodicity of the system is disrupted by the control law calculated here. The system seems to reach a new periodic state. The objective is the reduction of the lift coefficient (Figure 3b) throughout the 5.5 time units' horizon, but we are indirectly additionally looking for the reduction of the drag coefficient. Indeed, optimal control reduces the drag coefficient from 1.55 to 1.35.

4.2 2-degree of freedom optimal control

This section presents the results with alternative criteria, as we described in Section 3.2, hence the results are not directly comparable to the ones presented up to here. Throughout this section we will consider that the optimal control action magnitude is limited to at most 1, hence there are only 2 independent variables.

Firstly we consider the maximization of the downforce applied to the cylinder due to the fluid flow. This is expressed as the minimization of algebraic value of the lift coefficient throughout the simulation time, as in Equation (6). Following the principles set at Section 3, the bounds for the independent variables are:

$$\begin{bmatrix} -\pi/2 \\ 0 \end{bmatrix} \le x \le \begin{bmatrix} \pi/2 \\ T/2 \end{bmatrix}.$$

The optimal solution identified is $[-0.1035 \;\; 1.8196]^T$, for which the value of f is -111.8495. The optimization results are presented in Figure 4.

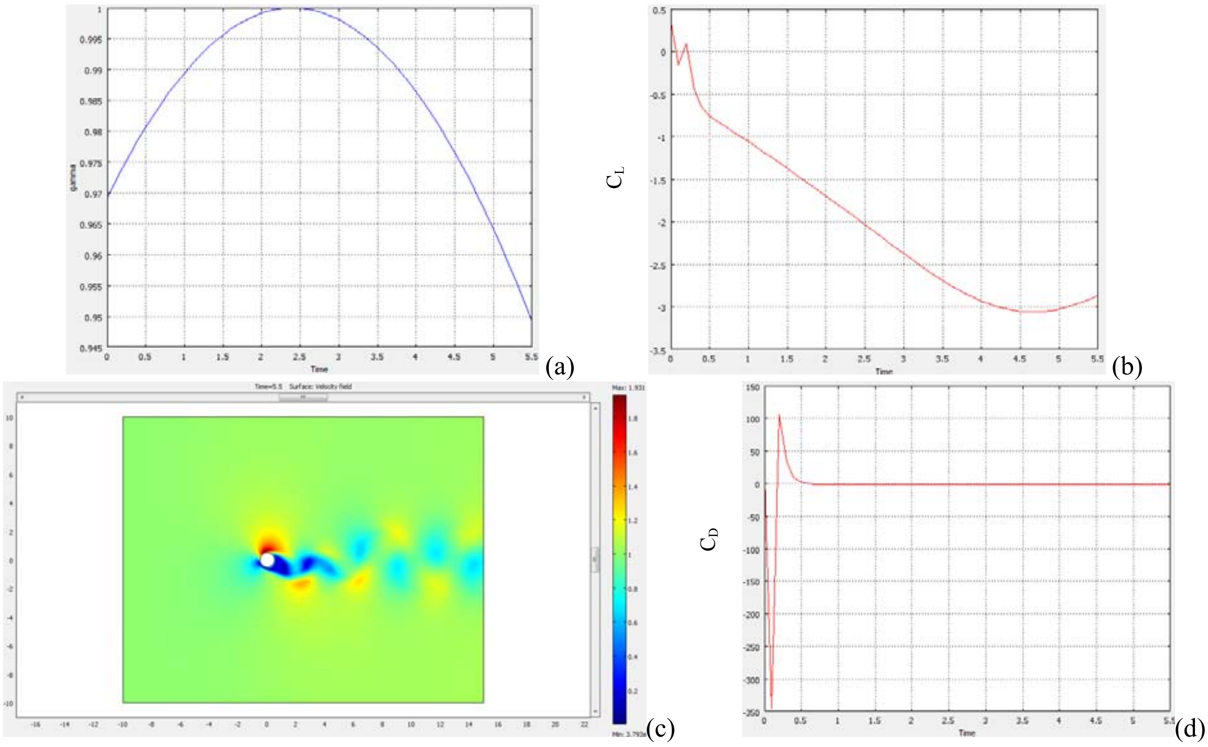


Figure 4: Optimization results for the 2-dof case A: optimal control action (a), resulting lift coefficient (b), velocity field profile after 5.5 time units (c) and temporal profile of the corresponding drag coefficient (d).

One can observe that the resulting flow profile in this case is significantly different than in other cases. This is because this case is formulated to maximize the vertical force applied on the cylinder. The lift coefficient profile is also very different than both the uncontrolled case (stationary cylinder) and the case of Section 4.1 (Figure 4b). Also, in this case the sinusoidal characteristics of the control law are conspicuous as one can see in Figure 4a.

As we discussed previously, the controlled system need not exhibit periodic behavior, or even if it does the period will generally not be the same as in the uncontrolled case. Indeed, in this case within the 5.5 time units of the simulation time we do not reach a periodic state, neither in the control law nor in the lift coefficient. It is expected that the periodicity of the control law may induce additional periodic phenomena of the lift coefficient; therefore the temporal profile of the lift coefficient will not be a simple harmonic, not even after it reaches periodic state.

In case the objective function is given by Equation (7) with $C_{L,target}$=0, the optimal solution identified is: $[-0.0223 \;\; 0.01684]^T$, for which the value of f is $3.024 \cdot 10^{-7}$, calculation which requires 41 function evaluations. The optimization results are presented in Figure 5.

One can observe that the control law is quasi-linear as in the first case and that the flow field is very similar to the one in case of stationary cylinder. This is because the mean value of the lift coefficient was already zero in the stationary case. The local optimizer located another solution that does achieve the desired. Indeed, the resulting optimal point located yields an objective function value in the order of $O(10^{-7})$, which for all practical purposes is zero.

This case highlights that the optimal is not necessarily unique. Even the *global* optimum need not necessarily be unique. In this case, it may prove that several control laws achieve a mean value of 0 for the lift coefficient over the prescribed dimensionless time horizon. Each of the local and global optima have an attractor as far as the optimization procedure is concerned. If an initial guess that is "close" to an optimum is used for optimization, the latter procedure is bound to find this optimum. The notion of "neighborhood" that defines what "close" is, depends on many things, including the topology of the problem around this optimum and the stability of the underlying physical system. However, since the procedure is gradient-based, it is not uncommon for the procedure to "jump" to a different region of parametric space (i.e. the space of the independent variables), especially in the first iterations and converge to a point different than expected, even if the expected point is the trivial solution.

Finally, we need to stress that the formulation of case B is quite general and can be used to track specific lift coefficients. This tracking need not necessarily be of a constant (zero or non-zero) value, but can in fact be a reference trajectory of a piecewise affine function (this case is quite common), or in general any desired lift coefficient profile.

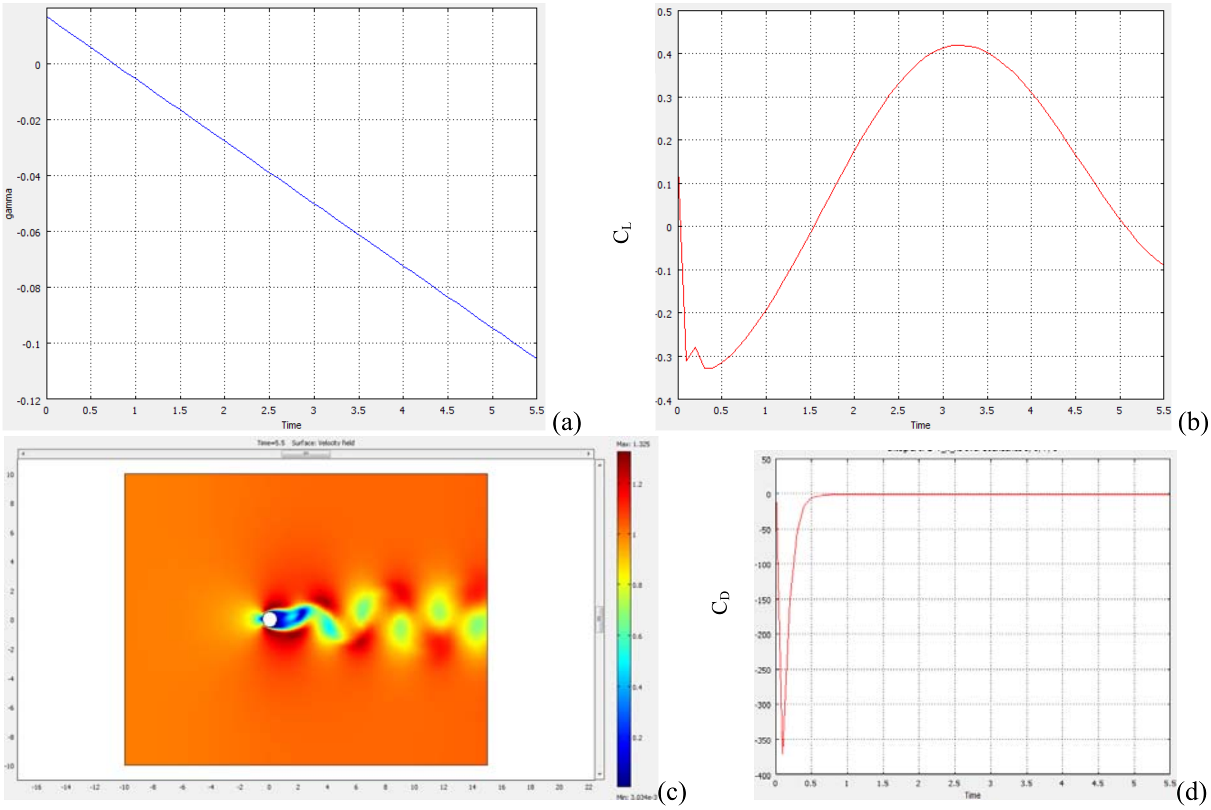


Figure 5: Optimization results for the 2-dof case B: optimal control action (a), resulting lift coefficient (b), velocity field profile after 5.5 time units (c) and temporal profile of the corresponding drag coefficient (d).

CONCLUSIONS

We have presented a framework for optimal control of systems exhibiting vortex shedding. We were concerned with wake formation behind a cylinder, for laminar flow with Reynolds number of 100. Multiple scenarios have been examined, expressing different objectives one may have for wake formation. We have considered that the angular speed of the cylinder can be controlled freely within certain limits. Our objective has been to minimize vortex shedding which would minimize the lift coefficient. The latter has been the objective function for our optimal control problem. Since wake formation is periodic, we have assumed that the optimal solution would also be periodic and could be expressed as a sinusoidal expression with degrees of freedom that are determined by the mathematical programming solution. Following our approach, the temporal profile for the cylinder angular speed is calculated offline and is considered set for a given Reynolds number. We also dealt with alternative criteria for vortex shedding systems. Namely, we considered minimizing the magnitude of the lift coefficient to minimize wake formation, to minimize its algebraic value so that there is a non-zero vertical force applied on the cylinder by the fluid and finally minimize the mean lift coefficient. We considered using 2 and 3 independent variables expressing more and less constrained cases and obtained efficient controllers.

Further work will examine the effect of Reynolds number to the optimal control law. Furthermore, the multimodality of the optimization problem will be explored, by employing multi-start techniques. Finally, we plan to address the minimization of the computational cost, by incorporating Model Order Reduction techniques and/or surrogate models for the substitution of the detailed model while preserving most of its accuracy.

REFERENCES

1. Graham, W. R., Peraire, J., & Tang, K. Y. (1999). Optimal control of vortex shedding using low-order models. Part I—open-loop model development. International Journal for Numerical Methods in Engineering, 44(7), 945-972.
2. Multiphysics, C. O. M. S. O. L. (2008). 3.5 a User's Guide. Comsol AB.
3. Bonis, I., Sarimveis, H., Koubogiannis, D.G. (2013). "Proper Orthogonal Decomposition-based reduced order modelling of vortex shedding", 5th International Conference on Experiments/Process/Systems Modelling/Simulation/Optimization (IC-EpsMsO).
4. Koubogiannis, D.G., Sarimveis, H., Bonis, I. (2014). "Development of a reduced order model for the numerical simulation of the laminar flow around a rotating cylinder", FLOW 2014: 9th Panhellenic Conference on Fluid Transport Phenomena.
5. Mathworks, I. (2007). Matlab Optimization Toolbox User's Guide. MathWorks, Inc. http://www. mathworks. com/access/helpdesk/help/pdf_doc/optim/optim_tb. pdf.
6. Li, Z., Navon, I. M., Hussaini, M. Y., & Le Dimet, F. X. (2003). Optimal control of cylinder wakes via suction and blowing. Computers & Fluids, 32(2), 149-171.
7. Homescu, C., Navon, I. M., & Li, Z. (2002). Suppression of vortex shedding for flow around a circular cylinder using optimal control. International Journal for Numerical Methods in Fluids, 38(1), 43-69.
8. Joe, W. T., Colonius, T., & MacMynowski, D. G. (2010). Optimized waveforms for feedback control of vortex shedding. In Active flow control II (pp. 391-404). Springer Berlin Heidelberg.